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Distance Measures of Pythagorean Fuzzy TOPSIS Approach for Online Food Delivery Apps

H. D. Arora*, A. Naithani, S. Gupta

Department of Applied Mathematics, Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida, India

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A B S T R A C T

The expansion of the online food delivery apps (OFDAs) around the globe has accelerated because of the sudden growing cases of the COVID-19 pandemic. OFDAs are quickly expanding in India, providing a huge number of chances for different OFDA platforms and creating a competitive market. There are several criteria and dimensions for OFDAs businesses to explore to keep with the frequently changing competitive market and achieve long-term success. A Pythagorean fuzzy set (PFS) is a powerful tool for dealing with uncertainty. Distance measure of PFS is a hot research topic and has real-life applications in many areas, such as decision making, medical diagnosis, patterns analysis, clustering, etc. The article aims to examine the results of the novel Pythagorean fuzzy distance measure strategy to select the best online app using TOPSIS method to select the best OFDAs. Firstly, all the axioms related to distance measures are proved for the proposed measures. The proposed work uses five distinct alternatives/options and four attributes/criteria in a fuzzy environment to deal with imprecise and conflicting information. The findings indicate that the proposed methodology is a more realistic way to choose the best OFDAs among others. Finally, a sensitivity analysis is used to determine whether the chosen alternative was the best option among the other components and to ensure that the TOPSIS technique results were accurate.

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NOMENCLATURE

OFDA	Online food delivery apps	FS	Fuzzy sets
MCDM	Multi-criteria decision making	IFS	Intuitionistic fuzzyzy sets
TOPSIS	Technique for order of preference by similarity to ideal solution	PFS	Pythagorean fuzzy sets
MD	Membership degree	NMD	Non-membership degree
PIS	Positive ideal solution	NIS	Negative ideal solution

1. INTRODUCTION

Making decision is undoubtedly one of the most fundamental activities of human beings. Decision making is the study of how decisions are made and how they can be made better. It is broadly defined to include any choice of alternatives and is of importance in many fields. It refers to identifying the optimum alternative or determining the ranking of alternatives.

MCDM is a complex decision-making method that incorporates both quantitative and qualitative elements. Several MCDM strategies and approaches have been suggested to to choose the most likely optimal options. As an augmentation to the fuzzy MCDM approach is suggested in this work, where the rankings of options versus attributes, and the weights of all criteria, are assessed in linguistic values represented by fuzzy numbers. MCDM models under fuzzy environment have been proposed by several researchers [1-5].

Zadeh [6] introduced the fuzzy concept for handling ambiguity in a better way. By assigning the MD to elements with respect to a set, a fuzzy set describes the state between "exist" and "does not exist." Atanassov [7, 8] suggested IFS which also included NMD $'\varrho(x)'$ along

*Corresponding Author Institutional Email: hdarora@amity.edu
(H. D. Arora)

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with $'\kappa(x)'$ satisfying $0 \leq \kappa(x) + \varrho(x) \leq 1$ and $\kappa(x) + \varrho(x) + \lambda(x) = 1$, where $\lambda(x)$ is the hesitancy function. As a result, it has been discovered that IFSs are better at representing human expressions such as perception, knowledge, and behaviour than FS. PFS was proposed by Yager [9, 10], in which the square sum of the MD and NMD is less than one. The main advantages of such extended FSs are that they use MD, NMD, and the degree of reluctance to represent ambiguous information. Later it was found that there were sets which were not satisfying the above condition hence a need to revise the sets were felt.

Several similarity and distance measures have been designed and analysed in the previous studies to quantify the similarity or difference between PFS. These measures have been effectively used to a variety of applications. Chen [11] established a distance for PFS based on the Minkowski distance measure and applied it to many real life problems such as Internet stock and reseaech and development project investment and various other practical situations. Lai et al. [12] presented and demonstrated the utility of numerous distance measures for PFS and Pythagorean fuzzy numbers. Wei and Wei [13] proposed ten different forms of PFS similarity measurements based on covariance. Ejegwa [14] applied distance measures for IFS, such as Hamming, Euclidean, normalized Hamming, and normalized Euclidean distances, as well as similarities to PFSs, multi-criteria and multi-attribute decision-making situations. Xiao and Ding [15] proposed a new measure using Jenson-Shannon divergence measure. Peng [16] and Adabitabar et al. [17] devised new Pythagorean distance and similarity measures with real life illustrations. Ejegwa [18] normalized the distance function introduced by Zhang and Xu [19] and validated the axiomatic definition of a metric for the updated version, which was missing in Zadeh’s work. Zhou and Chen [20] proposed new distance measure and applied it to real-life problems [21].

Boran and Akay [22] developed a biparametric similarity measure and applied to the approach to pattern recognition. Iqbal and Rizwan [23] considered the importance of intuitionistic fuzzy sets using similarity measure and its applications to decision making. Ejegwa and Agbetayo [24] introduced a new similarity-distance measure and applied it to decision making problem. A comparative analysis was also presented to validate the measure proposed. A modified PF correlation measure with application to decision making was suggested by Ejegwa et al. [25]. Lin et al. [26] gave some practical examples to highlight how the proposed directional correlation coefficient can be used in virus detection and in what way the suggested weighted directional coefficient of correlation can be used in cluster analysis.

In today's economy, due to the importance of quality and quantity of the product, supplier selection plays a significant role in procurement planning of each factory. A growing trend towards computerization and competition in supply chains results in uncertainty and quick variability that make the decisions difficult for both levels of retailers and manufacturers. Kaviyani-Charati et al. [27] novel approach to determine the optimal production and order quantities and prices with and without agile abilities. Cheraghalipour et al. [28] considered supplier selection framework for this industry and employed the best worst method (BWM) along with a well-known MCDM technique with the name of VIKOR. Shahsavar et al. [29] propsoed an efficient and robust decision-making framework for the concept of a green city and sustainable development goals to manage municipal plastic wastes. Sardi et al. [30] introduced a new approach in the field of port performance evaluation based on the components of greenness and intelligence. Ghoushchi et al. [31] proposed a novel approach to selecting the optimal landfill for medical waste using Multi-Criteria Decision-Making (MCDM) methods. For better considerations of the uncertainty in choosing the optimal landfill, the MCDM methods are extended by spherical fuzzy sets (SFS). Cheraghalipour et al. [32] developed a hybrid MCDM method and mixed integer linear programming (MILP) approach in order to evaluation of the returned products' collectors along with their ordered quantities. Fasihi et al. [33] proposed a novel mathematical model to maximize responsiveness to customer demand and minimize the cost of the fish closed-loop supply chain. A new three-phase model is presented by Valinejad et al. [34] to supply chain sustainability risks management. This model includes the failure mode and effects analysis phase for identifying and assessing all risks and classification them, fuzzy VIKOR phase for ranking critical risks, and management phase to deal with critical risks. The categorization of risks was conducted according to a new five-dimensional approach to sustainable progress, including environmental, economic, social, technical, and organizational aspects on various sectors of the supply chain. Afshar et al. [35]investigated dimensions of the cost of quality in a cold supply chain design such as the cost of quality related to suppliers and the cost of distribution service quality to close the problem to real-world conditions. Moreover, the quality of suppliers, manufacturers, and distributors was simultaneously considered throughout a supply chain with a new approach. Nozari et al. [36] suggested a model to locate warehouses and production centers and route vehicles for the distribution of medical goods to hospitals. The robust fuzzy method controled uncertain parameters, such as demand, transmission, and distribution costs. The effect of uncertainty using a neutrosophic fuzzy programming method showed that by increasing demand, the volume of medical goods exchanges and the number of vehicles used to distribute goods increase. Zahedi-Anaraki et al. [37] propsoed a modified benders decomposition algorithm for a last-mile network with flexible delivery options.

An ideal solution comprises of the optimal values of all criteria whereas a negative-ideal solution comprises of worst values of all criteria and the selection criteria for alternatives are based on Euclidean distance. The TOPSIS method is easy in implementation and has been applicable in the problems of selection and ranking of alternatives. MCDM methods are popular among the researchers in handling with decision- making problems to get the most reliable alternative. The TOPSIS method was developed by Hwang and Yoon [38] to solve decision-making problems. Using TOPSIS, we can conveniently determine the minimum distance between a positive ideal and a negative ideal solution, which supports choosing the best alternative. Several researchers used the TOPSIS method after its development for decision making and extended it to FSs, IFSs and PF environments [39-48].

According to statistics, India has 749 million mobile internet users till 2020. There's no doubt that mobile apps have become our live partners for anything and everything. Mobile apps have made our lives so much easier, from paying bills to ordering groceries over the phone. Food delivery apps on demand are handy and simple to use, and they offer tempting savings and faster delivery. The best food is delivered at the most affordable prices. Bangalore, Pune, Delhi, Gurgaon, Hyderabad, Chennai, and Mumbai are all hotspots for this trend of Indian cities. Therefore, goal of this article is to find a best online food delivering app based on certain criteria selected.

Research Gap

The study of PFS is recently gaining importance due to its wide application in situations involving ambiguity. It can easily be merged with MCDM techniques to solve real life problems. Many distance measures approaches have been suggested and applied to solve decision-making problems. Though the existing distance measures are somewhat significant, they possess some shortcomings in terms of accuracy and their alignments with the concept of IPFS, which needed to be strengthened to enhance reliable outputs. The study focusses on

- re-examines certain existing distance measures between PFS,
- offers an improved distance technique between PFSs,
- validate the proposed distance measures using axiomatic definition of distance measures,
- presents comparison analyses of the new distance technique in Pythagorean fuzzy environment,
- applies the new distance technique to determine some decision-making situations, and
- sensitivity analysis for assigning different weights by changing the criteria weights to those obtained by PF-distance measures to observe how much it would influence the final rankings of alternatives.

The rest of this article is organized as follows: The second section delivers many terminologies which will help evaluate performance of various app. The next section proposes a novel distance measure and its axioms are proved. In section 4, TOPSIS algorithm and the procedure to find the ranking of these apps are discussed. Section 5 presents the results and discussion. Finally, section 6 concludes the article.

2. PRELIMINARIES

Definition 2.1 [49]. Let $\mathcal{H}$ be defined as a Fuzzy set in non-empty set X then it is represented as:

$$\mathcal{H} = \{<x, \kappa_{\mathcal{H}}(x) \mid x \in X\} \quad (1)$$

where $\kappa_{\mathcal{H}}(\mathbb{x})$: X → [0, 1] is the MD of an element x in set X.

Definition 2.2 [7]. Let $\mathcal{H}$ be an IFS in X we can define:

$$\mathcal{H} = \{<x, \kappa_{\mathcal{H}}(x), \varrho_{\mathcal{H}}(x) \mid \forall\, x \in X\} \quad (2)$$

where $\kappa_{\mathcal{H}}(x)$: X → [0,1] and $\varrho_{\mathcal{H}}(x)$: X → [0,1]
$\kappa_{\mathcal{H}}(x)$ is the MD and $\varrho_{\mathcal{H}}(x)$ is the NMD such that $0 \le \kappa_{\mathcal{H}}(x) + \varrho_{\mathcal{H}}(x) \le 1$.

Definition 2.3 [9] A PFS is given as:

$$\mathcal{H} = \{<x, \kappa_{\mathcal{H}}(x), \varrho_{\mathcal{H}}(x) \mid \forall\, x \in X\} \quad (3)$$

and $\kappa_{\mathcal{H}}(x)$: X → [0, 1] and $\varrho_{\mathcal{H}}(x)$: X → [0, 1]
where, $\kappa_{\mathcal{H}}(x)$ is MD and $\varrho_{\mathcal{H}}(x)$ is the NMD such that:

$$0 \le \kappa_{\mathcal{H}}^2(x) + \varrho_{\mathcal{H}}^2(x) \le 1 \quad (4)$$

and

$$\lambda_{\mathcal{H}}^2(x) = 1 - \kappa_{\mathcal{H}}^2(x) - \varrho_{\mathcal{H}}^2(x) \quad (5)$$

where $\lambda_{\mathcal{H}}(\mathbb{x})$ called hesitancy or indeterminacy of PFS $\mathcal{H}$.

Definition 2.4 [50]. Assume
$\Delta_n = \{P = (p_1, p_2, \ldots, p_n): p_i \ge 0, \sum_{i=1}^{n} p_i = 0\}$ be a collection of n −complete probability distributions. For any probability distribution:

$$E(P) = -E_0 \sum P_{ij}\, In(P_{ij}) \quad (6)$$

where, $i = 1,2 \ldots n$; $j = 1,2, \ldots m$ and E_0 is the entropy constant calculated by $\frac{1}{In\, m}$.

3. DISTANCE MEASURES FOR PFS

3. 1. Existing Distance Measures

Definition 3.1 [19]. Distance between PFSs $\wp$ and $\mathbb{Q}$, is defined as:

$$D_{ZX}(\wp, \mathbb{Q}) = \frac{1}{2}\sum_{i=1}^{n}\{|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)| + |\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)| + |\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)|\} \quad (7)$$

Definition 3.2 [51]. Normalized Hausdorff distance between PFSs $\wp$ and $\mathbb{Q}$, is defined as:

$$D_{HY}(\wp,\mathbb{Q}) = \frac{1}{n}\left\{\sum_{i=1}^{n} \max\left\{\begin{matrix}\left|\kappa_{\wp}^2(\mathrm{x_i}) - \kappa_{\mathbb{Q}}^2(\mathrm{x_i})\right|, \\ \left|\varrho_{\wp}^2(\mathrm{x_i}) - \varrho_{\mathbb{Q}}^2(\mathrm{x_i})\right|\end{matrix}\right\}\right\} \quad (8)$$

Definition 3.3 [18]. Modified Zhang and Xu distance between PFSs $\wp$ and $\mathbb{Q}$ is defined as:

$$D_{MZX}(\wp,\mathbb{Q}) = \frac{1}{n} D_{ZX}(\wp,\mathbb{Q}) \quad (9)$$

where $D_{ZX}(\wp,\mathbb{Q})$ denotes Zhang and Xu [19] distance measure demonstrated in Equation (7).

Definition 3.4 [11]. Chen's distance measure between PFSs $\wp$ and $\mathbb{Q}$, is defined as:

$$D_C(\wp,\mathbb{Q}) = \frac{1}{2n}\sum_{i=1}^{n}\left\{\left|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)\right|^{\beta} + \left|\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)\right|^{\beta} + \left|\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)\right|^{\beta}\right\}^{\frac{1}{\beta}} \quad (10)$$

where β is a distance parameter, satisfying $\beta \geq 1$. If $\beta = 1$, it reduces to the Hamming distance. If $\beta = 2$, it reduces to Euclidean distance.

Definition 3.5 [52]. Given a finite universe $\mho$, distance measure between PFSs $\wp$ and $\mathbb{Q}$ is defined as:

$$D_{MP}(\wp,Q) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right|}{\kappa_{\wp}^2(x_i)+\kappa_{\mathbb{Q}}^2(x_i)+\varrho_{\wp}^2(x_i)+\varrho_{\mathbb{Q}}^2(x_i)} \quad (11)$$

where $\wp = \{\langle \mathrm{x_i}, \kappa_{\wp}(\mathrm{x_i}), \varrho_{\wp}(\mathrm{x_i})\rangle | \mathrm{x_i} \in \mho\}$ and $\mathbb{Q} = \{\langle \mathrm{x_i}, \kappa_{\mathbb{Q}}(\mathrm{x_i}), \varrho_{\mathrm{Q}}(\mathrm{x_i})\rangle | \mathrm{x_i} \in \mho\}$

3. 2. Proposed Distance Measure for PFSs

Firstly, we recall the axiomatic preposition of divergence for Pythagorean fuzzy sets.

Proposition 1. Let $\wp$, $\mathbb{Q}$, $\mathcal{R}$ $\in$ PFS (X) where X is a non-empty set. The distance measure between $\wp$ and $\mathbb{Q}$ is a function that satisfies

(D1) $0 \leq Div(\wp,\mathbb{Q}) \leq 1$

(D2) $Div(\wp,\mathbb{Q}) = 0 \Leftrightarrow \wp = \mathbb{Q}$.

(D3) $Div(\wp,\mathbb{Q}) = Div(\mathbb{Q},\wp)$

(D4) If $\mathcal{R}$ is a PFS in X and $\wp \subseteq \mathbb{Q} \subseteq \mathcal{R}$, then $Div(\wp,\mathbb{Q}) \leq Div(\wp,\mathcal{R})$ and $Div(\mathbb{Q},\mathcal{R}) \leq Div(\wp,\mathcal{R})$.

It is important to consider the weight of each elements as in decision making process, factors typically have distinctive importance, so they should be given different weights. Taking weights into considerations, we proposed two novel distance measures between $\wp$ and $\mathbb{Q}$ as follows:

Assume $\wp,\mathbb{Q} \in$ PFS (X) where $X = \{x_1, x_2 \ldots, x_n\}$ then:

$$D_{PFS}(\wp,\mathbb{Q}) = 1 - \frac{3}{n}\left[\sum_{i=1}^{n}\left\{\frac{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}-1}{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}+1}\right\}\right] \quad (12)$$

$$D_{WPFS}(\wp,\mathbb{Q}) = 1 - \frac{3}{n}\left[\sum_{i=1}^{n}\omega_i\left\{\frac{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}-1}{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}+1}\right\}\right] \quad (13)$$

where, $\lambda_{\wp}(x_i) = \sqrt{1 - \kappa_{\wp}^2(x_i) - \varrho_{\wp}^2(x_i)}$ and $\lambda_{\mathbb{Q}}(x_i) = \sqrt{1 - \kappa_{\mathbb{Q}}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)}$; ω is the weight vector of $x_i (i = 1,2,\ldots,n)$, with $\omega_i \in [0,1]$, $i = 1,2,\ldots,n$, $\sum_{i=1}^{n}\omega_i = 1$. If we take $\omega_i = 1$, then then $D_{WPFS}(\wp,\mathbb{Q}) = D_{PFS}(\wp,\mathbb{Q})$.

Theorem 3.1. The proposed measures specified in Equations (12) and (13) are valid measures of PFS.

Proof. All four criteria for a distance measure are satisfied by the proposed distance measures listed below:

(D1) $0 \leq D_{PFS}(\wp,\mathbb{Q}), D_{WPFS}(\wp,\mathbb{Q}) \leq 1$

Proof. For $\mathrm{D_{PFS}}(\wp,\mathbb{Q})$: As all the values of MD and NMD lies between 0 and 1, hence we can say that:

$0 \leq \left|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)\right| \leq 1$,

$0 \leq \left|\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)\right| \leq 1$,

$0 \leq \left|\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)\right| \leq 1$. Hence,

$$0 \leq \left|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)\right| \leq 3$$

$$\Rightarrow 0 \leq \frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)\right|\right\} \leq 1$$

$$\Rightarrow 0 \leq 1 - \frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i) - \kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i) - \varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i) - \lambda_{\mathbb{Q}}^2(x_i)\right|\right\} \leq 1 \quad (14)$$

$$\Rightarrow 2 \leq 2^{1-\frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right\}} \leq 1$$

$$1 \leq 2^{1-\frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right\}} - 1 \leq 0$$

Also,

$$\Rightarrow 3 \leq 2^{1-\frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right\}} + 1 \leq 2 \quad (15)$$

From (14) and (15), we have:

$$\frac{1}{3} \leq \frac{2^{1-\frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right\}}-1}{2^{1-\frac{1}{3}\left\{\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right\}}+1} \leq 0$$

$$0 \leq 1 - \frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}-1}{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}+1}\right\}\right] \leq 1$$

$$0 \leq 1 - \frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}-1}{2^{1-\frac{1}{3}\left[\left|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^2(x_i)\right| + \left|\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^2(x_i)\right| + \left|\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^2(x_i)\right|\right]}+1}\right\}\right] \leq 1$$

$\Rightarrow 0 \leq \mathrm{D_{PFS}}(\wp,\mathbb{Q}) \leq 1$.

Thus, $0 \leq \mathrm{D_{PFS}}(\wp,\mathbb{Q}) \leq 1$.

Measure $\mathrm{D_{WPFS}}(\wp,\mathbb{Q})$ can be proved similarly.

(D2) $D_{PFS}(\wp,\mathbb{Q})=0 \Leftrightarrow \wp = \mathbb{Q}$ and $D_{WPFS}(\wp,\mathbb{Q})=0 \Leftrightarrow \wp=\mathbb{Q}$.

Proof: For $D_{PFS}(\wp,\mathbb{Q})$: We consider two PFS $\wp$ and $\mathbb{Q}$ in $X=\{x_1,x_2 \dots,x_n\}$,
Let $\wp = Q$, then $\kappa^2_\wp(x_i)=\kappa^2_\mathbb{Q}(x_i)$, $\varrho^2_\wp(x_i)=\varrho^2_\mathbb{Q}(x_i)$ and $\lambda^2_\wp(\mathrm{x_i})=\lambda^2_\mathbb{Q}(\mathrm{x_I})$ which implies, $|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|=0$, $|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|=0$ and $|\lambda^2_\wp(\mathrm{x_i})-\lambda^2_\mathbb{Q}(\mathrm{x_i})|=0$.
Therefore,

$2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}-1=1$ and

$2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}+1=3$

$$\Rightarrow \frac{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}-1}{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}+1}=\frac{1}{3}$$

$$\Rightarrow \frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}+1}\right\}\right]=1$$

$$\Rightarrow 1-\frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}+1}\right\}\right]=0$$

$\Rightarrow D_{PFS}(\wp,\mathbb{Q})=0$.

If $D_{PFS}(\wp,\mathbb{Q})=0$, this implies

$$\frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}+1}\right\}\right]=1$$

$$\frac{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}-1}{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}+1}=\frac{1}{3}$$

Let $2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}=x$

$\frac{x-1}{x+1}=\frac{1}{3}$.

Therefore,

$3x-3=x+1 \Rightarrow \mathrm{x}=2$.

$\Rightarrow 2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}=2$
$\Rightarrow |\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|=0$, $|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|=0$ and $|\lambda^2_\wp(\mathrm{x_i})-\lambda^2_\mathbb{Q}(\mathrm{x_i})|=0$. Therefore $\kappa^2_\wp(x_i)=\kappa^2_\mathbb{Q}(x_i)$, $\varrho^2_\wp(x_i)=\varrho^2_\mathbb{Q}(x_i)$ and $\lambda^2_\wp(\mathrm{x_i})=\lambda^2_\mathbb{Q}(\mathrm{x_i})$. Hence $\wp=\mathbb{Q}$.
Measure $D_{WPFS}(\wp,\mathbb{Q})$ can be proved similarly.
(D3) $D_{PFS}(\wp,\mathbb{Q})= D_{PFS}(\mathbb{Q},\wp)$ and $D_{WPFS}(\wp,\mathbb{Q})= D_{WPFS}(\mathbb{Q},\wp)$
Proof for the above property is self-evident and direct.
(D4) If $\mathcal{R}$ is a PFS in X and $\wp\subseteq\mathbb{Q}\subseteq\mathcal{R}$, then $D_{PFS}(\wp,\mathbb{Q})\le D_{PFS}(\wp,\mathcal{R})$ and $D_{PFS}(\mathbb{Q},\mathcal{R})\le D_{PFS}(\wp,\mathcal{R})$.
Proof. If $\wp\subseteq\mathbb{Q}\subseteq\mathcal{R}$, therefore for $x_i\in \mathrm{X}$, we get

$0\le\kappa^2_\wp(x_i)\le\kappa^2_\mathbb{Q}(x_i)\le\kappa^2_\mathcal{R}(x_i)\le 1$,

$1\ge\varrho^2_\wp(x_i)\ge\varrho^2_\mathbb{Q}(x_i)\ge\varrho^2_\mathcal{R}(x_i)\ge 0$,

$0\le\lambda^2_\wp(x_i)\le\lambda^2_\mathbb{Q}(x_i)\le\lambda^2_\mathcal{R}(x_i)\le 1$. That implies

$|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|\le|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|$;

$|\kappa^2_\mathbb{Q}(x_i)-\kappa^2_\mathcal{R}(x_i)|\le|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|$;

$|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|\le|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|$;

$|\varrho^2_\mathbb{Q}(x_i)-\varrho^2_\mathcal{R}(x_i)|\le|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|$;

$|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\le|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|$;

$|\lambda^2_\mathbb{Q}(x_i)-\lambda^2_\mathcal{R}(x_i)|\le|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|$

From the above we can write,

$$|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\le|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\zeta^2_P(x_i)-\zeta^2_R(x_i)|+|\eta^2_P(x_i)-\eta^2_R(x_i)| \Rightarrow$$
$$2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}-1\le 2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}-1 \tag{16}$$

and

$$\mathbf{2}^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}+\mathbf{1}\le 2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}+1 \tag{17}$$

Dividing (16) by (17), we have:

$$\frac{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}-1}{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}+1}\le\frac{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}+1}\Rightarrow$$
$$1-\frac{3}{n}\left[\sum_{i=1}^{n}\left\{\frac{2^{1-\frac{1}{3}\{|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|\}}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathbb{Q}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathbb{Q}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathbb{Q}(x_i)|]}+1}\right\}\right]\ge 1-\frac{3}{\mathrm{n}}\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\frac{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}-1}{2^{1-\frac{1}{3}[|\kappa^2_\wp(x_i)-\kappa^2_\mathcal{R}(x_i)|+|\varrho^2_\wp(x_i)-\varrho^2_\mathcal{R}(x_i)|+|\lambda^2_\wp(x_i)-\lambda^2_\mathcal{R}(x_i)|]}+1}\right\}\right]\Rightarrow$$

$D_{PFS}(\wp,\mathcal{R})\le D_{PFS}(\wp,\mathbb{Q})$. Similarly, $D_{PFS}(\wp,\mathcal{R})\le D_{PFS}(\mathbb{Q},\mathcal{R})$

Similar proofs can be made for $\mathrm{D_{WPFS}}(\wp,\mathcal{R})\le \mathrm{D_{WPFS}}(\wp,\mathbb{Q})$ and $\mathrm{D_{WPFS}}(\wp,\mathcal{R})\le\mathrm{D_{WPFS}}(\mathbb{Q},\mathcal{R})$.

4. PRACTICAL APPLICATION OF PFS USING TOPSIS APPROACH

The following are the steps for proposed PFS TOPSIS approach [36]
Step 1. Construct decision matrix (DM) for PFS as follows:
Let there be n possibilities from the available options $\mathfrak{F}=\{\mathfrak{F}_1,\mathfrak{F}_2,\dots,\mathfrak{F}_n\}$. The possible criteria set can be written as $\mathfrak{C}=\{\mathfrak{C}_1,\mathfrak{C}_2,\dots,\mathfrak{C}_m\}$ with m options. Let us denote $D=\left[d^k_{ij}\right]_{m\times n}$ be a set of k-decision-makers or experts of PFSs value $d_{ij}=(\kappa_{ij},\varrho_{ij})$ is structured. Here, κ_{ij} and ϱ_{ij} are the MD and NMD of the alternatives $\mathfrak{F}_i$ satisfying the criteria $\mathfrak{C}_j$. The PFSs index $\lambda_{\mathrm{ij}}=\sqrt{1-\kappa^2_{\mathrm{ij}}-\varrho^2_{\mathrm{ij}}}$ displays the hesitation index of the alternative $\mathfrak{F}_i$ with respect to the criteria $\mathfrak{C}_j$.

Step 2. Normalize DM for PFS as follows:
Normalize the fuzzy DM based on benefit and cost criteria by interchanging MD with NMD and vice versa. In case of cost criteria; however, benefit criteria remain unchanged.
Step 3*:* Determination of weights for the criteria:
Initially weights of the criteria is taken as 0.37, 0.3, 0.23, and 0.1 for calculations. Later, sensitivity analysis has been done to validate our results.
Step 4*:* Compute PIS and NIS:
Find PIS and NIS for each criteria. This method divides assessment criteria into benefit (B) and cost (C). PIS and NIS can be constructed using PFSs and the traditional TOPSIS approach as:

$$\tilde{A}^{+} = \{r_1^{+}, r_2^{+}, \dots, r_n^{+}\} = \begin{Bmatrix} \left(\max_i \left(r_{ij}\right)/j \in B\right), \\ \left(\left(\min_i (r_{ij}) \;/\; j \in C\right)\right) \end{Bmatrix} \tag{18}$$

$$\tilde{A}^{-} = \{r_1^{-}, r_2^{-}, \dots, r_n^{-}\} = \begin{Bmatrix} \left(\min_i \left(r_{ij}\right)/j \in B\right), \\ \left(\left(\max_i (r_{ij}) \;/\; j \in C\right)\right) \end{Bmatrix} \tag{19}$$

Step 5: Calculate the divergence measures from the positive ideal and negative ideal solutions using propsoed mesures suggested in (12) and (13)

$$S_{WPFS}^{+} = D(A_i, \tilde{A}^{+}) = 1 - \left[\sum_{i=1}^{n} \left\{\frac{2^{1-\frac{1}{3}\{|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^{2+}(x_i)| + |\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^{2+}(x_i)| + |\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^{2+}(x_i)|\}}-1}{2^{1-\frac{1}{3}\{|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^{2+}(x_i)| + |\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^{2+}(x_i)| + |\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^{2+}(x_i)|\}}+1}\right\}\right] \tag{20}$$

And

$$S_{WPFS}^{-} = D(A_i, \tilde{A}^{-}) = 1 - \frac{3}{n}\left[\sum_{i=1}^{n} \left\{\frac{2^{1-\frac{1}{3}\{|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^{2-}(x_i)| + |\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^{2-}(x_i)| + |\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^{2-}(x_i)|\}}-1}{2^{1-\frac{1}{3}\{|\kappa_{\wp}^2(x_i)-\kappa_{\mathbb{Q}}^{2-}(x_i)| + |\varrho_{\wp}^2(x_i)-\varrho_{\mathbb{Q}}^{2-}(x_i)| + |\lambda_{\wp}^2(x_i)-\lambda_{\mathbb{Q}}^{2-}(x_i)|\}}+1}\right\}\right] \tag{21}$$

Step 6: Calculate the relative closeness to the positive ideal solution
For each decision-maker, the relative closeness coefficient of each alternative with respect to PFSs ideal solution is determined as:

$$\psi_i = \frac{D(A_i, \tilde{A}^{-})}{D(A_i, \tilde{A}^{+}) + D(A_i, \tilde{A}^{-})} = \frac{S_i^{-}}{S_i^{+} + S_i^{-}} \tag{22}$$

where $\psi_i \in [0,1], i = 1,2, \dots, n$.
The highest value of ψ_i indicates the preferred better the estimation of the available options.

4. 1. A Case Study With the growing popularity of smartphones, we can now have anything we need with only a few touches. This holds true for food as well. Food can be ordered online and delivered to our homes in minutes or hours, depending on the quantity of order and location. Several large organizations and startups are working hard to improve meal delivery services. In India, there are many meal delivery services that are competing to provide excellent service and cuisine. Most of them are limited to a single city, but a few have grown in popularity and are now available in many cities, providing excellent food delivery service. These meal delivery smartphone applications are vying to provide better service. Some of these meal delivery applications also include live tracking of the food delivery person, allowing us to keep track of our food and ensure that we never miss it. In this competitive environment, our objective is to find the best online food delivery app based on few criteria with the help of distance measure for Pythagorean fuzzy sets using TOPSIS approach. For that matter, we have chosen five top rated online food delivery apps viz., $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ and $\mathfrak{F}_5$ as alternatives and four criteria viz., Well designed app ($\mathfrak{C}_1$), offers ($\mathfrak{C}_2$), delivery time ($\mathfrak{C}_3$) and price ($\mathfrak{C}_4$), Five leading online food delivery apps are to be evaluated by the decision-maker under the above four criteria in the following steps:
Step 1: Construct a decision matrix for each alternative according to each decision maker in terms of PFSs which is presented in Table 1.
Step 2: Based on this data, these apps required to be ranked and the best app needs to be determined. In MADM, the second step is to classify the considered problem in benefit and cost criteria. Benefit criteria are those criteria for which higher values are desired and cost criteria are those for which lower values are desired. In the considered case study, $\mathfrak{C}_1$ and $\mathfrak{C}_2$ are benefit criteria whereas $\mathfrak{C}_3$, and $\mathfrak{C}_4$ are cost criteria. The normalized data is presented in Table 2.
Step 3: We identify the fuzzy PIS and fuzzy NIS.
The subsequent values are presented in Table 3
Step 4: We find the measures values from PIS and NIS using Equations (20) and (21) for the measure and depicted in Tables 4 and 5, respectively.
Step 5: Relative closeness coefficient using Equation (24) can be found and shown in Table 6.
Analysing the ranking of alternatives, we rank these food delivery apps based on how close they are to one another. From the above table it is evident that F1 is the best food delivery app, and they are categorized as $F1 \succ F5 \succ F4 \succ F3 \succ F2$. Figure 1 depicts the ranking of the alternatives.

4. 2. Sensitivity Assessment If decision-makers arrive at different rankings for the available options, the output of getting optimum alternative remain unsolved. To eliminate ambiguity regarding the best options in terms of

TABLE 1. Rating values of DM in terms of PFS

	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
$\mathfrak{F}_1$	(0.81, 0.57)	(0.36, 0.63)	(0.64, 0.31)	(0.75, 0.57)
$\mathfrak{F}_2$	(0.59, 0.72)	(0.25, 0.75)	(0.25, 0.92)	(0.52, 0.84)
$\mathfrak{F}_3$	(0.28, 0.63)	(0.29, 0.53)	(0.39, 0.56)	(0.37, 0.61)
$\mathfrak{F}_4$	(0.47, 0.60)	(0.33, 0.67)	(0.46, 0.35)	(0.47, 0.59)
$\mathfrak{F}_5$	(0.26, 0.42)	(0.82, 0.48)	(0.64, 0.72)	(0.79, 0.51)

TABLE 2. Rating values of DM in terms of PFS

	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
$\mathfrak{F}_1$	(0.81, 0.57)	(0.36, 0.63)	(0.31, 0.64)	(0.57, 0.75)
$\mathfrak{F}_2$	(0.59, 0.72)	(0.25, 0.75)	(0.92, 0.25)	(0.84, 0.52)
$\mathfrak{F}_3$	(0.28, 0.63)	(0.29, 0.53)	(0.56, 0.39)	(0.61, 0.37)
$\mathfrak{F}_4$	(0.47, 0.60)	(0.33, 0.67)	(0.35, 0.46)	(0.59, 0.47)
$\mathfrak{F}_5$	(0.26, 0.42)	(0.82, 0.48)	(0.72, 0.64)	(0.51, 0.79)

TABLE 3. PIS and NIS for each criterion

	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
FPIS	(0.81, 0.42)	(0.82, 0.48)	(0.31, 0.64)	(0.51, 0.79)
FNIS	(0.26, 0.72)	(0.25, 0.75)	(0.92, 0.25)	(0.84, 0.37)

TABLE 4. Separation measures for fuzzy PIS

		$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
For FPIS	$\mathfrak{F}_1$	0.11192	0.06528	0.07666	0.031995
	$\mathfrak{F}_2$	0.09669	0.06084	0.03944	0.023893
	$\mathfrak{F}_3$	0.07767	0.06227	0.06427	0.022982
	$\mathfrak{F}_4$	0.08923	0.06391	0.06717	0.024812
	$\mathfrak{F}_5$	0.07679	0.10000	0.05611	0.03333

TABLE 5. Separation measures for fuzzy NIS

		$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
For FNIS	$\mathfrak{F}_1$	0.07679	0.08967	0.039446	0.024326
	$\mathfrak{F}_2$	0.10157	0.10000	0.076666	0.030564
	$\mathfrak{F}_3$	0.11401	0.08229	0.050559	0.026317
	$\mathfrak{F}_4$	0.11115	0.09294	0.040806	0.025800
	$\mathfrak{F}_5$	0.09669	0.06084	0.059857	0.022982

TABLE 6. Ranking result obtained from TOPSIS approach

	S_i^+	S_i^-	R_i	Ranking
$\mathfrak{F}_1$	0.7855956	0.8273251	0.512936	1
$\mathfrak{F}_2$	0.83434	0.768393	0.4794267	5
$\mathfrak{F}_3$	0.8295969	0.7951123	0.4893874	4
$\mathfrak{F}_4$	0.8161437	0.7969739	0.4940582	3
$\mathfrak{F}_5$	0.8003206	0.8197145	0.5059857	2

decision-makers, distinct expert values are combined by allocating a priority value to each expert so that $\rho_i > 0$ and $\sum_{i=1}^{n} \rho_i = 1$. Using these weight vectors, the distance measure of each expert is consolidated, and the overall estimated values of the alternatives are derived, as shown in Table 6:

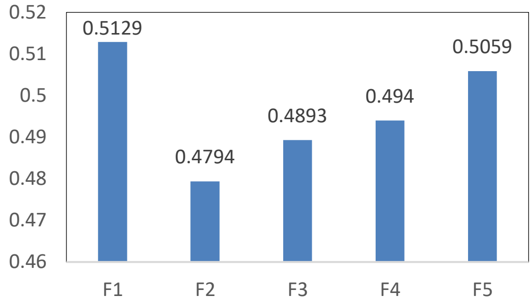

Figure 1. Ranking of alternative

$$\eta_i^+ = \sum_{k=1}^{s} \rho_k C_{ij}^+ \tag{23}$$

$$\eta_i^- = \sum_{k=1}^{s} \rho_k C_{ij}^- \tag{24}$$

Also,

$$\psi_i = \frac{\eta_i^-}{\eta_i^+ + \eta_i^-} \tag{25}$$

where $\psi_i \in [0,1], i = 1,2, \dots,5$.

The relative closeness and ranking for each app is summarized in Table 7.

We opted to slightly adjust the original initial weights by examining four different situations to give accurate analysis in testing, the sensitivity of the results to changes in input parameters, especially decision makers weights. The suggested models provide the same output as (OFDA) is the best choice in all the cases. Therefore, it suggests that the proposed technique is accurate and reliable.

5. COMPARATIVE ANALYSIS

The proposed distance measures are compared with current measures based on the numerical scenarios presented to establish the supremacy. Table 8 shows a comparative analysis of the distance measures.

From the numerical results presented in Table 8, it was noticed that the findings achieved by projected distance measures are analogous with outcomes of prevailing measures.

TABLE 7. Relative closeness and ranking for each app

Case I: $\rho_1 = 0.42, \rho_2 = 0.28, \rho_3 = 0.1, \rho_4 = 0.2$

Alternatives	Distance Measures		ψ_i	Rank	Best App
	η_i^+	η_i^-			
$\mathfrak{F}_1$	0.786019	0.8225029	0.511340	1	
$\mathfrak{F}_1$	0.826383	0.7726748	0.483206	5	
$\mathfrak{F}_1$	0.834847	0.7893657	0.485998	4	$\mathfrak{F}_1$
$\mathfrak{F}_1$	0.820162	0.7883032	0.490096	3	

$\mathfrak{F}_1$	0.796325	0.8210934	0.507656	2	
Case II: $\rho_1 = 0.5, \rho_2 = 0.2, \rho_3 = 0.2, \rho_4 = 0.1$					
Alternatives	**Distance Measures**		$\boldsymbol{\psi_i}$	**Rank**	**Best App**
	η_i^+	η_i^-			
$\mathfrak{F}_1$	0.779924	0.833366	0.51656	1	
$\mathfrak{F}_1$	0.827927	0.774125	0.483208	5	
$\mathfrak{F}_1$	0.830985	0.790586	0.487543	4	$\mathfrak{F}_1$
$\mathfrak{F}_1$	0.815180	0.794910	0.493705	3	
$\mathfrak{F}_1$	0.810574	0.815299	0.501452	2	
Case III: $\rho_1 = 0.15, \rho_2 = 0.2, \rho_3 = 0.4, \rho_4 = 0.25$					
Alternatives	**Distance Measures**		$\boldsymbol{\psi_i}$	**Rank**	**Best App**
	$\boldsymbol{\eta_i^+}$	$\boldsymbol{\eta_i^-}$			
$\mathfrak{F}_1$	0.773333	0.83475	0.519097	1	
$\mathfrak{F}_1$	0.843926	0.761807	0.474429	5	
$\mathfrak{F}_1$	0.818319	0.808894	0.497104	4	$\mathfrak{F}_1$
$\mathfrak{F}_1$	0.806764	0.818130	0.503497	3	
$\mathfrak{F}_1$	0.790957	0.81901	0.508712	2	
Case IV: $\rho_1 = 0.15, \rho_2 = 0.2, \rho_3 = 0.25, \rho_4 = 0.40$					
Alternatives	**Distance Measures**		$\boldsymbol{\psi_i}$	**Rank**	$\boldsymbol{\mathfrak{F}_1}$
	η_i^+	η_i^-			
$\mathfrak{F}_1$	0.774838	0.826680	0.516185	1	
$\mathfrak{F}_1$	0.836340	0.764922	0.477699	5	
$\mathfrak{F}_1$	0.823901	0.804017	0.493892	4	
$\mathfrak{F}_1$	0.811707	0.809064	0.499184	3	
$\mathfrak{F}_1$	0.780905	0.822432	0.512950	2	

TABLE 8. Comparative Analysis of the Distance Measures

Comparison	$\mathfrak{F}_1$	$\mathfrak{F}_2$	$\mathfrak{F}_3$	$\mathfrak{F}_4$	$\mathfrak{F}_5$
Distance Measure [19]	0.6827	0.1819	0.3641	0.4148	0.5818
Normalized Hausdorff distance [49]	0.6827	0.1819	0.3641	0.4148	0.5818
Chen's distance measure [11]	0.6740	0.1995	0.3733	0.4255	0.5654
Distance Measure [52]	0.7162	0.1257	0.3338	0.4429	0.6380
Proposed Distance	0.5129	0.4794	0.4893	0.4940	0.5059

6. CONCLUSIONS

Distance measures are an effective tool for measuring uncertainty using Pythagorean fuzzy sets. In this article, an innovative approach is used to measure the performance of online food delivery apps (OFDA), ensuring that the results are accurate every time. The salient feature of the proposed distance measures is their efficiency in distinguishing PFS with high hesitancy. The proposed distance measures satisfy the useful properties in the proven theorems. Comparison between alternatives or characteristics that uses fuzzy membership tackles the uncertainty and erroneous judgement. By using the TOPSIS approach, this paper presents an innovative study in MCDM in a fuzzy TOPSIS environment. Because of its capacity to accommodate decision makers' hazy opinions and perceptions, the TOPSIS method is the best way for tackling MCDM challenges. This novel model assists decision makers in thematically making error-free decisions, regardless of the multi-criteria field. The concept of TOPSIS can be applied to solve real life problems in fuzzy environments, which have uncertainty problems associated with them.

From the study, it is observed that the novel distance measures for PFS give reliable outputs compared to the existing ones and, hence, can suitably handle multi criteria decision making effectively.

TOPSIS approach is rational and understandable, and the computation process are straightforward. However, this method presents certain drawbacks. One of the problems attributable to TOPSIS is that it can cause the phenomenon known as rank reversal. Rank reversal occurs when a decision maker, in the process of selecting an option from a set of choices, is confronted with new alternatives that were not thought about when the selection process was initiated. It depends on the relationship between this new alternative and the old ones under each criterion. Therefore, modifications in the algorithm of TOPSIS approach will certainly resolve this issue.

We believe, the proposed distance measures will find its serviceability in new avenues of application. Future study in this direction includes

- Parametric generalizations of similarity measures for PFS
- Application of proposed distance measures fro interval valued PFS,
- Development of new MCDM approaches and comparing them with the suggested approach
- Utility of distance measures to intuitionistic fuzzy sets, Fermatean fuzzy sets, soft sets, rough sets etc.
- Applications to entropy-distance measures in decision making.

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Persian Abstract

چکیده

گسترش برنامه‌های ارسال غذای آنلاین (OFDA) در سراسر جهان به دلیل افزایش ناگهانی موارد همه‌گیری COVID-19 سرعت گرفته است. OFDA به سرعت در هند در حال گسترش است و فرصت های زیادی را برای پلتفرم های مختلف OFDA فراهم می کند و بازار رقابتی ایجاد می کند. معیارها و ابعاد مختلفی برای کسب‌وکارهای OFDA وجود دارد که باید آن‌ها را بررسی کنند تا با بازار رقابتی که اغلب در حال تغییر است و به موفقیت بلندمدت دست پیدا کنند. مجموعه فازی فیثاغورث (PFS)یک ابزار قدرتمند برای مقابله با عدم قطعیت است. اندازه گیری فاصله PFS یک موضوع تحقیقاتی داغ است و کاربردهای واقعی در بسیاری از زمینه ها مانند تصمیم گیری، تشخیص پزشکی، تجزیه و تحلیل الگوها، خوشه بندی و غیره دارد. هدف این مقاله بررسی نتایج استراتژی جدید اندازه گیری فاصله فازی فیثاغورث است. بهترین برنامه آنلاین را با استفاده از روش TOPSIS انتخاب کنید تا بهترین OFDA را انتخاب کنید. ابتدا، تمام بدیهیات مربوط به اندازه گیری های فاصله برای اقدامات پیشنهادی اثبات شده است. کار پیشنهادی از پنج گزینه / گزینه متمایز و چهار ویژگی / معیار در یک محیط فازی برای مقابله با اطلاعات نادقیق و متناقض استفاده می کند. یافته‌ها نشان می‌دهد که روش پیشنهادی روشی واقعی‌تر برای انتخاب بهترین OFDA در میان سایر روش‌ها است. در نهایت، از یک تحلیل حساسیت برای تعیین اینکه آیا جایگزین انتخاب شده بهترین گزینه در میان سایر مؤلفه‌ها بوده و برای اطمینان از دقیق بودن نتایج تکنیک TOPSIS استفاده می‌شود.