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A Constraint Programming Approach to Solve Multi-skill Resource-constrained Project Scheduling Problem with Calendars

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ABSTRACT

The multi-skill resource constrained project scheduling is an important and challenging problem in project management. Two key issues that turn this topic into a challenging problem are the assumptions that are considered to approximate the model to a real-world problem and exact solution approach for the model. In this paper, we deal with this two issues. To consider real-word situations, we take into account calendars specifying time intervals during which the resources are available. We proposed a constraint programming approach to solve the problem exactly. The problem with and without resource calendars are modeled with mathematical programming (MP) and constraint programming (CP). In addition, the performance of CP approach is evaluated by comparing Time-Indexed Model (TIM) and Branch and Price (B&P) approaches. Computational results show that the proposed approach can efficiently solve real-size instances.

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1. INTRODUCTION

Project scheduling is one of the most influential subjects in project management. Project scheduling consists of finding a scheduling pattern for all the activities and the constraint satisfaction simultaneously in order to optimize an objective function of the problem. In order to consider the realistic features which often occur in project scheduling problem in the real world and approximate the actual problem, many assumptions are taken into consideration in resource constrained project scheduling problem (RCPSP) [1-6]. One of these assumptions which is having an influential effect on executing a project is resource calendar constraint. The calendar constraint plays a significant role in project scheduling problems and has many applications in real-life cases. Recently, some of the investigations focus on project scheduling problems with calendars. Franck et al. [7] addressed the problem of scheduling the activities of a project subject to calendar constraints. They presented efficient algorithms for computing the earliest and latest start and completion times of activities. Kreter et al. [8] extended the RCPSP by the concept of break-calendars by considering the possibility of the absence of renewable resources. Ahmadpour and Ghezavati [9] proposed a new mathematical model for the RCPSP under uncertainty in which the resources are not available at any time.

Each resource in RCPSP masters only one skill to execute an activity and each activity needs one skill to be executed, while in a real situation each resource has one or more skills. When the resources are multi-skill, not only it is important to choose which of the resources to be assigned to each activity but also the skill with which they contribute to each activity [10]; that is called Multi-skill Resource-Constrained Project Scheduling Problem (MS-RPSP). Considering the resources as multi-skill, leads the MS-RCPSP to become more complex than RCPSP [11]. Approaching this problem having another perspective we could consider MS-RCPSP as an extension of RCPSP, because the formulation of RCPSP can be used in order to illustrate a MS-RCPSP, but MS-RCPSP is more complex than RCPSP.

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One of the challenging features in project scheduling problems is finding the optimum allocation of the resources. Using the multi-skill resources brings about advantages such as flexibility in scheduling that causes improvement in productivity, decreasing the idle time of resources, and reducing the labor cost. These advantages caused the researchers to use multi-skill resources in their research. The primary investigations in this field were conducted by Bellenguez-Morineau and Neron [12].

The solution techniques that are used in MS-RCPSP are divided into two categories, heuristic and meta-heuristic approaches besides exact approaches. The articles that have proposed the former category for MS-RCPSP are as follows: Avramidis et al. [13] studied an agent scheduling problem in which each agent has a selected number of skills and the agents are differentiated by the set of call types they can do. The goal is to minimize the total cost of the project. Cai and Li [14] proposed a multi-criteria model to formulate the staff scheduling problem with multiple skills and solved the problem with a meta-heuristic approach. Wongwai and Malaikrisanachalee [15] firstly prioritized the tasks then they allocated the resources to the tasks regarding the priorities created at the former step. In case of inadequate resources an activity cannot be commenced, therefore, the activity uses the resource of other activities with lower priorities. Walter and Zimmermann [16] deals with a multi-skill project scheduling problem and consider the skills as hierarchical levels, and the objective is to minimize the average project team size and satisfy the requirements of the project and the departments. The most important articles which proposed an exact solution for MS-RCPSP are as follows: In the survey by Heimerl and Kolisch [17] the skills are considered as hierarchical levels, therefore, every resource can have a high level of skill that helps them to be able to execute activities faster or in higher quality relative to the other resources. Furthermore, in the investigation by Bellenguez-Morineau and Néron [12] also the skills are considered in hierarchical levels. A review paper was published in the field of MS-RCPSP by De Bruecker et al. [18] that would be useful for the readers.

In this paper, staffing and scheduling problems with multi-skill resources are considered and the resource calendars are added to the problem. As mentioned before, calendar constraints are taken into account in RCPSP because the resource calendar has a significant role in real applications of scheduling problems. This assumption could be more important for MS-RCPSP because in MS-RCPSP each activity needs the set of skills to be executed and these skills must be assigned to one human resource that masters one of the required skills. For each activity, if any of its required resources become unavailable, that activity cannot start until all of its required resources become available. So the MS-RCPSP with calendar constraint is more complex than RCPSP with this assumption. Another contribution of the current research in solution approach is a CP model developed for solving the MS-RCPSP with and without Calendar constraints. Besides the developed CP models, two mathematical models are provided for comparison with the results of CP models.

The rest of this paper is organized as follows: In section 2 the problem is defined mathematically and sets, notations, parameters, and assumptions are explained. The third and fourth sections respectively describe the MP and CP models. In section 5 the experimental results are presented and two models were compared. Finally, in section 6 we conclude the paper and present our perspective of future studies.

2. PROBLEM DESCRIPTION

To describe the problem first consider a project that contains the set A of activities, where each activity has to be processed without preemption to complete the project. The project resources master the set k of skill and each activity needs also one or more skills to be executed, the project is represented by an activity on node network $N = (A, E)$ where E represents the direct successor of the activities, Activities *0* and *n+1* are dummy and represent the beginning and the end of the project, the network is acyclic and the weight of an $\text{arc}(i, j) \in E$ is equal to the processing time of activity i. For executing each of the activities, the set of skills that are needed by that activity must be simultaneously accomplished, the number of these operations are fixed at the project and all of the operations must be executed during the project.

To the best of our knowledge, all researches in MS-RCPSP consider the resources available at any time during the project execution; however, at the real condition sometimes some resources could not be accessed, hence considering the resource calendar will be worthwhile. Off-work days including weekends and holidays are having a significant effect on the project execution time due to the fact that human resources would not be available during these days. The consideration of a resource calendar is necessary for many operative MS-RCPSP applications in which manpower are required.

In fact, each resource has its own specific time calendar during the project. Therefore taking the time calendar into account for each of the resources at the scheduling phase plays a significant role in controlling the budget and executing the project according to the scheduling plan. To diminish the disruptive effect of the off days, it is highly desired to distinguish the interruptible and non interruptible activities. For instance, non-interrupting task such as heating which takes 5 work days cannot be commenced on Thursday morning; hence, to be assured of having it executed continuously, it has to be started on days between Monday and Thursday.

In this paper mathematical equalities similar to Montoya et al. [19] have been proposed, however, some inequalities have been added to the model for defining calendar for each resource. After defining the mathematical model, the substructure of constraint programming has been drawn out and the CP model is proposed.

2. 1. Problem Statement The sets and parameters that use for the models are listed below:

A	:	The set of activities, i=0,1,…,n+1
R	:	The set of resources, j=1,2,…,$\|R\|$
K	:	The set of skills, k=1,2,…,$\|K\|$
T	:	The set of time periods, t=1,2,…,$\|T\|$
p_i	:	Processing time of activity i
ES_i	:	The earliest start of activity i
LS_i	:	The latest start of activity i
E_i	:	The set of direct successors of activity i
$b_{i,k}$	:	The number of workers mastering skill k needed for performing activity i
RS_j	:	The set of skills that resources j master it
AS_i	:	The set of skills that activity i need it

$TM_{j,t}$ and $w^i_{j,t}$ are considered to define the resource calendar. $TM_{j,t}$ is a binary variable and shows the availability of the resources in each time period and $w^i_{j,t}$ determines the feasibility of assigning resource j to activity i at time period t. For instance, resource 2 at 5th and 6th days of each week is on vacation, so for the t= 1, 2, 3, 4 and 7, $TM_{2,t}$ equals 1. Now, consider that we are at the third day of the project and activity 3 with a processing time of 4 is possible to be executed at time 3, due to the assumption of non-preemptive activities, assignment of resource 2 to activity 3 at time 3 is impossible, because after 2 working days the resource is unavailable, so the value of $w^3_{3,2}$ equals 0.

All resources which are used during the project are renewable which means that, under consideration of the resource calendar, each resource is available after completion of the operation which is assigned to the resource. Requirements of the resources are expressed in a matrix $AS_{i,k}$ and also $O_{j,k}$ and $AL_{i,k,j}$ define the all operation of the project and all doable assignments, respectively. To attain the solution, all the operations must be taken into account but some rows could be optional in the matrix of $AL_{i,k,j}$.

2. 2. Assumptions The list of assumptions that must be enumerated to complete the explanation of our framework are:

- The problem consists of a set of non-pre-emptive activities.
- All resources are non-renewable and each of them could not execute more than one operation at a time.
- The set of skills that are needed by each activity must be executed simultaneously.
- Each resource has a specific calendar and the availability of each resource during the project are different.
- All parameters and variables are considered an integer.

3. MATHEMATICAL PROGRAMMING

We define four binary decision variables: $w^i_{j,t}$ determines the feasibility of assigning resource j to activity i at time t, z_i^t which takes the value 1 if an activity i starts at time t, x_{ij}^t states if worker j starts activity i at time t and y_{ijk} which takes the value 1 if worker j uses skill k to perform activity i. The main elements of the model is as follows:

- **Precedence relationships**

The relation between the activities are represented by Equation (1).

$$\sum_{t=ES_i}^{LS_i} tz_i^t + p_i \le \sum_{t=ES_j}^{LS_j} tz_j^t \quad \forall i \in A \quad , \quad \forall j \in E_i \tag{1}$$

- **Resource constraint**

Equation (2) states that if each resource is available in the specific time window, it can start an activity at most once, during the time window. Equation (3) ensures that each resource can execute at most one activity at each time period.

$$\sum_{t=ES_i}^{LS_i} x_{ij}^t \le 1 \qquad \forall i \in A \quad , \quad \forall j \in R \tag{2}$$

$$\sum_{i=1}^{n} \sum_{d=\max\{ES_i, t-p_i+1\}}^{\min\{LS_i, t\}} x_{ij}^d \le 1 \quad \forall j \in R, \ \forall t \in T \tag{3}$$

- **Resource calendar**

Constraints (4-6) guarantees that each resource can carry out each activity just during its specified calendar.

$$\left\lceil \frac{\sum_{d=t}^{t+p_i-1} TM_{j,d}}{p_i} \right\rceil = w_{i,j}^t \quad \forall t \in \mathrm{T},\ \forall i \in A,\ \forall j \in R \tag{4}$$

$$w_{i,j}^t * x_{i,j}^t \le z_i^t \quad \forall t \in \mathrm{T},\ \forall i \in A,\ \forall j \in R \tag{5}$$

$$w_{i,j}^t * x_{i,j}^t + 1 \ge z_i^t + \sum_{\forall k \in AS_{i,k}} y_{i,j,k} \quad \forall t \in \mathrm{T},\ \forall i \in A,\ \forall j \in R \tag{6}$$

- **Simultaneously execution**

A set of skills that was required by an activity was fulfilled by different resources, Equation (7) ensures that all of these skills are simultaneously executed.

$$\sum_{\forall j \in R} x_{i,j,t} = z_i^t * \sum_{\forall k \in K} AS_{i,k} \quad \forall t \in \mathrm{T},\ \forall i \in A \tag{7}$$

- **Resource allocation**

Constarints (8) and (9) state the requirements of each activity are satisfied by allocating the resources to them and each resource executes at most one skill of activity.

$$\sum_{\forall j \in R} y_{i,j,k} = b_{i,k} \quad \forall i \in A,\ \forall k \in RS_j \tag{8}$$

$$\sum_{t=ES_i}^{LS_i} x_{i,j}^t = \sum_{\forall k \in RS_j} y_{i,j,k} \quad \forall i \in A,\ \forall j \in R \tag{9}$$

- **Objective function**

Equation (10) aims to minimize the makespan of the project.

$$\min \quad \sum_{t=ES_{n+1}}^{LS_{n+1}} t * z_{n+1}^t \tag{10}$$

4. CONSTRAINT PROGRAMMING

CP formation started in 1970s and development stages were passed in parallel in US, Germany, and Australia; additionally Sabin and Freuder [20] disregarding previous investigations devised an approach in order to define and solve the problem using a specific language. It is essential to mention that the term "state" which had been used in the paper, defining and declaring the problem does not mean to give the linear mathematic model to the software to be solved; ironically, it means to define the problem in a different (expressive language) way. Expressive language is the most important advantage of CP optimizer and this language is based on the notion of interval variables [21]. The CP optimizer comes up in the artificial intelligence area that investigates Constraint Satisfaction Programming (CSP) and Logic Programming [21]. In the objective function in CP at decision problem, an optimal solution is not of interest; instead, the main goal in the decision problem is to find a feasible solution. CP cannot be counted as a solution algorithm or even a solution methodology; even though CP presents the solution methodology the solution cannot be limited to it; thus, CP cannot be considered as a software to solve the model. To present a correct definition for CP one can say it is a concept like LP and MILP. In CP the term CSP is called for decision variables, in CSP the goal is to find a feasible solution or allocate a series of values of the variables in order to satisfy the constraints and substructures. The significant techniques of CP are propagation and search. Consistency is a key factor for constraint propagation, the consistency warrants every value in a domain to be consistent with every constraint [21]. CP has been proven to be very efficient in solving scheduling problems, the basis of CP is to decrease the domain of the variables, as long as every variable takes a unique value this algorithm uses every constraint in order to reduce the domain of variables, this procedure is fully described in Figure 1. Choosing the variable and branching at the decision tree could have a substantial impact on the computational time.

Combining the MP and CP are useful to achieve the solution faster. The common method to combine CP and mathematical model is to firstly solve the linear relaxation problem and after that use the solution of the relaxation problem for reducing the domain of variables or helping to choose a better branch.

A valuable point about CP is that it can solve every non-linear problem or any problem with discrete variables or with non-convex solution space. The further study in the CP area in scheduling problem [22] are worth to mention.

We propose the CP algorithm as shown in Figure 2.

Sets and parameters are defined through lines 1 to 7 and decision variables are defined through lines 8 to 11. The objective function is to minimize the makespan of the project which was illustrated in line 12. In lines 13-19, the set of skills that was required by an activity was simultaneously executed and activity allocation to resources were considered. The algorithm of alternative and calendar constraints are shown in Figures 3 and 4. Lines 20-22 define the relationship between the activities. Lines 23-26 guarantee that each resource is only able to execute the activity in its time calendar. Lines 27-29 ensure that every resource can execute at most only one skill of activity at a specific time.

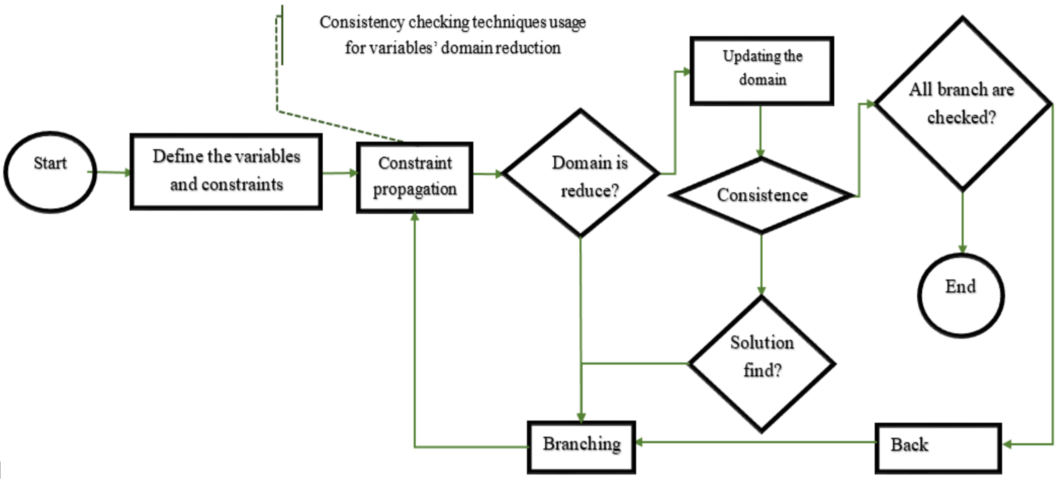

Figure 1. Procedure of CP optimization

```
Input :
1)R  : Set of resources
2)A  : Set of activity
3)K  : Set of skill
4)O  : set of Operation
5)AL : set of Allocation
6)Act _Skill : Set of skill that activity needed
7)Res _skill : Set of skill that resource master it
Variable :
8)act -time[A]
9)act -skill -time[O]
10)act -skill -res -time[AL]
11)workers[R]
12)Minimize Max (EndOf (Act -time[A]))
S.T
13)     For i ← 1to i ← |A|
14)          For j ← 1to j ← |O|
15)            Synchronize(i, j) Act -skill -time[j] where | j.task == r
16)            Alternative(act -skill -time[j]|where j.task == i , (awhere a.ops == o))
17)          End
18)     End
19)End

20)For i, j ← 1to i, j ← |A| | A[i][j] == 1
21)   EndBeforStart(i, j, 0)
22)End

23)For a ← 1to a ← |AL|
24)   ForbidStart(act -skill -res -time(a),calendar(a.res)
25)   ForbidEnd(act -skill -res -time(a),calendar(a.res)
26)End

27)Forj ← 1to j ← |R|
28)   NoOverlap(workers(j))
29)End
```

Figure 2. The CP algorithm

5. EXPERIMENTAL RESULTS

In this section, the results of computational experiments are presented. Acquiring the demanded tools to make the desired comparison, the two models are tested and now it is feasible to determine which can be the more successful. The models were both run on an Intel Core i5 duo processor with 8 GHz RAM. The Mathematical model using integer programming was coded in optimization programming language and solved by CPLEX 12.6.

The CP model uses a constraint programming method, it was solved by the Constraint Programming Optimizer included in ILOG optimizers. Since the CP algorithm works upon a set of parameters, Taguchi method is used for optimizing the parameters

To the best of our knowledge, no investigation has considered the calendar constraints for MS-RCPSP. Therfore, for having a better comparison, the problems are solved with MP and CP and the results of solutions were compared.

```
n = |RS_r|
r_j
= set of resources which can execute the activity a
x is the objective fuction
1) If interval variable a is present
2) c=0;
3) r1 = r*;
4) For i=r1 to i=rn
5) IF x(r_{i+1}) is better than x(r_i)
 6) c ← x(r_{i+1})
7) else
8) c ← x(r_i)
9) End If
10) End for
11) End If
```

Figure 3. The algorithm of Alternative

```
A : set of activity
BT : Break time
AV : available
R : set of resources
D_t : set of break time in t'thposition
For j ← 1 to |R|
  For t ← 1 to T
      AV[j][t] ← 1
  end
end
For  j ← 1 to |R|
 For  z ← to |D|
    For t  ← min{D_z} to max {D_z}
        AV[j][t] ← 0
     end
  end
end
For  i ← 1 to |A|
   For j ← 1 to |R|
       If (Σ_{d=t}^{t+p_i−1} TM_{jdt}) / p_i ==1
       endIf
   end
end
```

Figure 4. The algorithm of calendar constraint

In the next step, for further validation of the solution approach, it is needed to equalize the assumptions of our work with the problem investigated by Montoya et al. [19]. The calendar constraints are relaxed; then the acquired results from the solution of the relaxed problem (which is solved by CP algorithm) were compared with Montoya et al. [19]'s results.

5. 1. Input Data In order to evaluate the proposed methods of MS-RCPSP, the set of instances which was proposed by Montoya et al. [19] are used in this study. The instances involved between 20 and 62 activities, 2 and 15 skills, and 2 and 19 resources. The 271 related instances are divided into three groups (similar to Montoya et al. [19]), the groups are shown in Table 1.

5. 2. Taguchi Method To design an experiment and optimize the parameters based upon the above-specified conditions Taguchi method was used. The statistical approach used here named as Taguchi method considers three conditions, (a) larger the better (b) smaller the better (c) On-target, minimum variation. Taguchi method suggests the matrix of experiments and can get maximum information from a minimum number of experiments and also the best level of each parameter can be acquired for an objective function. Signal-to-noise ratios are used to compute the response of the empirical test. The configuration of the CP package did not employ the default settings and we selected the best options based on Taguchi method, for performing this method three parameters at four levels have been considered as shown in Table 2.

In the orthogonal array the levels corresponding to each control factor are similarly integrated with other factor level; hence, the number of runs to study three parameters at four levels diminish to only 16 instead of 3^4=81 runs. The experimental tests were conducted in a total of sixteen runs to calculate the SNR which is summarized in Table 3.

Figure 5 shows the main plot based upon average values of SNR, the horizontal axis shows the levels and the vertical axis shows the signal of noise ratio, the larger the SNR value, the better the parameter. For the result which is presented in Figure 6, values of the parameters are set to A= low, B=100, C= 1.3.

TABLE 1. Instance category

	Activity	**Resource**	**Skill**
Group 1	20-51	5-14	2-8
Group 2	32-62	5-19	9-15
Group 3	22-32	4-15	3-12

TABLE 2. Levels for control factors used in the experiment

Control Factor	**Level**			
	1	**2**	**3**	**4**
A: inference level	low	basic	medium	extended
B: restart fail limit	50	80	100	150
C: restart growth factor	1.05	1.1	1.3	1.7

TABLE 3. Experimental design with SNR

Ex. Run	**A**	**B**	**C**	**SNR**	**MEAN**
1	low	50	1.05	61.9936	1258
2	low	80	1.1	75.7902	6159
3	low	100	1.3	55.9591	628
4	low	150	1.7	79.7577	9725
5	basic	50	1.1	67.7406	2438
6	basic	80	1.05	64.5833	1695
7	basic	100	1.7	71.3098	3677
8	basic	150	1.3	72.3985	4168
9	medium	50	1.3	68.5885	2688
10	medium	80	1.7	74.9108	5566
11	medium	100	1.05	76.9032	7001
12	medium	150	1.1	70.1571	3220
13	extended	50	1.7	75.5499	5991
14	extended	80	1.3	78.7111	8621
15	extended	100	1.1	72.2726	4108
16	extended	150	1.05	80.3206	10376

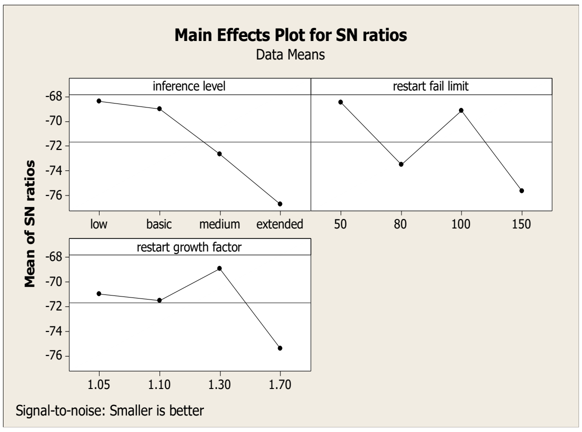

Figure 5. Effect of control factors on CPU time

5. 3. Computational Results For having a better insight into how the CP and MP models are performed, 9 instances that are randomly selected among three groups proposed by Montoya et al. [19] were considered. The results of solving these instances under resource calendar assumption by MP and CP are taken into consideration and data are shown in Table 4. According to the results, it goes without saying that the CPU time of CP is much less than MP. The major point is the number of variables each technique had taken into account; which according to the outcomes the average number of used variables by MP is 57 times bigger than CP. (note: in CP due to the use of “Alternative interval variables”, some of the variables do not activate during solving the problem). In general, the net number of the problems in which their optimal solutions had been achieved are more in CP relative to MP and this hypothesis is also valid in all our nine instances. Table 4 shows that the number of the achieved optimal solutions for MP and CP are five and nine respectively.

The prominent point about the CP model is that when the calendar constraints are considered, the CPU time for solving the problem is reduced; with that in constraint propagation technique, the bigger the number of the constraints, the faster the algorithm in reducing the domain of the variables and reaching the solution. This fact is presented in Table 5.

5. 4. Computational Results by Relaxing the Calendar Constraint According to compare our results with the results achieved by Montoya et al. [19], we initially removed the calendar constraint then modeled the problem by CP. The comparison results are shown in Table 6. It can be conspicuously observed in Tables 6 and 7 that CP model outperforms the B&P and TIM in terms of obtained optimal solutions and also a shorter total average CPU time than the TIM and B&P. It is obviously seen that average CPU time of the CP is shorter than other two methods, except for group 2 it is slightly longer than average CPU time of the B&P.

TABLE 4. Comparison of the solutions obtained through MP and CP

Problem number	No. of activities	No. of resources	No. of skills	**CPU time** (in seconds)		No. of variables		No. of constraints	
				CP	MP	CP	MP	CP	MP
1	20	13	3	23.25	126.65	287	13212	549	18797
2	22	11	3	12.20	112.28	235	14534	444	26043
3	22	7	3	13.54	65.87	180	11048	334	21572
4	27	10	3	0.34	97.35	397	17537	774	31901
5	32	15	12	0.36	221.59	474	24923	1076	48751
6	32	16	12	0.46	no answer	552	24987	1081	48876
7	51	10	3	2.26	no answer	612	45383	1332	81602
8	62	10	9	46.76	no answer	699	56161	1383	99202
9	62	19	9	135.66	no answer	1328	68254	3055	112301

TABLE 5. Comparison of the solutions obtained by CP with and without assumption of calendars

Problem number	No. of activities	No. of resources	No. of skills	**CPU time** (in seconds)		No. of variables		No. of constraints	
				CP^1	CP^2	CP^1	CP^2	CP^1	CP^2
1	22	11	3	85.87	12.20	235	235	112	444
2	22	7	3	174.54	13.54	180	180	94	334
3	27	10	3	0.42	0.341	397	397	162	774

CP^1: CP without assumption of calendars & CP^2: CP with assumption of calendars

TABLE 6. The obtained result of TIM, B&P and CP

	Optimal solution		
	CP	**B&P**	**TIM**
Group 1	84%	43.6%	21.81%
Group 2	77%	66.19%	64.78%
Group 3	63%	51.11%	15.55%

TABLE 7. Average CPU time of TIM and B&P and CP

	Average CPU time (in seconds)		
	CP	**B&P**	**TIM**
Group 1	49.9	148.95	1420.81
Group 2	83.3	57.09	737.32
Group 3	66.75	283.02	1569,21

6. CONCLUSIONS

In this paper, we proposed a constraint programming approach for the multi-skill resource constrained project scheduling problem. In order to improve the problem in terms of consideration of real-world factors, the resource calendars is taken into account. The results showed that the proposed CP outperformed mathematical programming approach. This model can use specialized constraints, such as the synchronize constraint, that can shorten the time of achieving the solution. In addition, the comparison between CP and branch-and-price algorithm with no calendar constraint was made and the results show that the CP model is faster than the branch-and-price algorithm in finding the optimal solution. The CP approach allows us to deal with complex problems and consider the problem concepts by its substructures successively and reach the solution in an aptly short time.

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Persian Abstract

چکیده

زمانبندی پروژه منابع محدود چندمهارته یکی از مسائل چالشی و مهم در مدیریت پروژه است. دو موضوع کلیدی این مسئله را چالش برانگیز می‌کند، مفروضاتی که برای تقریب مدل به یک مسئله واقعی در نظر گرفته می‌شود و نیز حل دقیق مسئله. در این مقاله، به این دو موضوع می‌پردازیم. بدین منظور، برای درنظرگیری شرایط دنیای واقعی، تقویم کاری منابع را در مدل وارد می‌کنیم تا روزهای کاری و غیرکاری منابع در مسئله لحاظ شود. همچنین از برنامه‌ریزی محدودیت برای حل دقیق مدل و استخراج جواب بهینه استفاده می‌شود. مسئله با و بدون درنظرگیری تقویم منابع با استفاده از برنامه ریزی‌ریاضی و همچنین برنامه‌ریزی محدودیت مدلسازی می‌شود. همچنین، عملکرد رویکرد برنامه ریزی محدودیت با رویکرد TIM و رویکرد B&P مورد ارزیابی قرار می‌گیرد. نتایج محاسباتی نشان می‌دهد که رویکرد پیشنهادی می‌تواند به طور موثری نمونه‌های با اندازه واقعی را حل نماید.