



## Response of Rubcrete Continuous Deep Beams under Sinusoidal Loads

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### ABSTRACT

Continuous deep beams (CDBs) are the most used members in constructions with highly exposing to different types of dynamic loads. It is well known that; the concrete is a brittle material and has a weak resistance to energy absorption. Using scrapped tire rubber enhances the concrete energy absorption for sustainability purposes. Timoshenko beam theory has been used to solve CDBs subjected to sinusoidal load and has been adopted for verification of numerical results of ANSYS APDL V.15.0. Seven concrete mixes have been simulated with different types and amounts of aggregate – rubber replacements. Several parameters have been studied like replacing type, percentages, shear span of beam to depth ratio ( $a/h$ ) and load intensity. It was found that Timoshenko beam theory can be used for harmonic loading CDBs. Furthermore, replacement in general provided more ductility due to rubber elasticity property. Gravel replacement by 45% has the larger displacement values among the other types. Also, it has been noted that, the sensitive of concrete deep beams towards  $a/h$  ratio stills considerable for harmonic loads, i.e. minimizing the ratio leads to decrementing the deflection wave amplitudes.

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### NOMENCLATURE

a	Beam shear span	$w_c$	Concrete weight
b	beam width	$W_n$	Natural frequency
c	Damping factor	$z$	Studied point after pending (or depth of beam)
$c^2$	Correction factor	<b>Greek Symbols</b>	
E	Concrete elastic modulus	$\epsilon_x$	Strain due to x-axis
F(t)	Applied harmonic load (point load)	$\gamma_{zx}$	Shear strain due to z-x axes
$f'c$	Concrete compressive strength	$\sigma_x$	Concrete stress towards x-axis
G	Shear modulus	$\tau_{zx}$	Shear stress toward z-x axes
I	Moment of inertia of the section	$\psi$	Rotation angle
K	Beam stiffness	<b>Abbreviations</b>	
M	Moment of beam	ACI	American code institute
m	Beam mass (Kg)	CDBs	Continuous deep beams
Q	Beam shear	FR	Fine replacement
t	Applied load time.	GR	Gravel replacement

## 1. INTRODUCTION

Concrete continuous deep beams (CDBs) are widely used members as a load distributor (due to its high rigidity and stiffness) in many constructions like bridges, high rise buildings, girders and tanks (as shown in Figure 1) [1-5]. American Concrete Institute (ACI 318-19 code) specified

two conditions for considering the concrete beam as a deep, which are, the clear span to total depth ratio does not exceed 4, or the concentrated load lies within the distance  $2h$  from the supporting face [6]. Concrete deep beams largely exposed to dynamic loads in different intensities and since the concrete is a brittle material, then researches nowadays towards to enhance the concrete

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energy absorption to dynamic loads. The published researches confirmed that, replacing scrapped tire rubber by a percentage of aggregate (or even adding it into the mix) provides an excellent energy absorption (but decreases the mechanical properties of concrete) [7-16]. The enhancement in impact energy of rubberized concrete (rubcrete) mixes reaches to (138-185-300-396%) for 5, 10, 15, 20% volumetric sand replacement, While (150-204-326-426%) for the same percentages of volumetric gravel replacement [17]. Too many types of dynamic loads may the CDBs exposed to, such as impact, seismic and moving vehicles load.

The simple form of harmonic load may occur by unbalanced rotating machines in building. Also, it may cause by hydrodynamic pressure which is generated due to propeller at the stern of a ship or by inertial effects in reciprocating machinery, and this types is more complex. It is worth to mention that the harmonic wave may come in different types which are: fundamental wave, 3rd harmonic wave and the distorted wave.

CDBs may exposed to harmonic loads at bridges due to vehicle movement and that will cause sin or cosine wave load on it. Adding rubber into concrete mix leads mainly to develop the concrete energy absorption of the bridges and enhance the overall dynamic properties. It can be noted from the next section that, there is no cutting-edge studying results gives us an indication about the CDBs response under harmonic loading. So, this paper consists of three parts, the first one is derived in a theoretical solution for analyzing the CDBs under harmonic loads. While the second part involved using the theoretical solution for checking the accuracy of numerical analysis. The third part studied the effect of adding rubber to concrete mix on the behavior of CDBs using ANSYS V.15.0 software program.

## 2. LITERATURE REVIEW

Generally, the response of continuous concrete deep beams has not investigated by the past researches especially for rubcrete mixes while it can be found some researches about an empirical equations to solve the seismic loads [18-21]. Numerical solutions by different software programs is interested with the reinforced concrete beams [22]. There are also some theoretical studies about concrete beams which exposed to harmonic loads [23-27] and plate foundation with regarding to Winkler model of subgrade reaction [28-30]. Through literature [25], the cracked and un-cracked concrete cantilever beam was investigated under the effect of harmonic load. It was found that, crack existing near to the fixed end support of the cantilever beam decreases the Eigen frequency compared with crack existing away from the fixed support. Also, displacement will be more

for cracked beam compared to healthy beam, because of reduction in stiffness. Chen and Song [27] introduced a theoretical study for solving deep beams which were exposing to distributed harmonic load. Finite element (FE) method (By Matlab software [31]) has been used for certificating the theoretical solution. It was noticed an excellent matching between the derived equation and the FE results. Continuous deep beams have been studied in many articles with static load [32-35], repeated and cyclic loads on simply supported deep beams [34, 36], rubberized continuous deep beams were statically tested [37, 38], and rubberized deep beams under static loads [39]; but, there was no investigation under the effect of harmonic loads. Therefore, solving the CDBs under the effect of harmonic loads has not been studied yet. Furthermore, the effect of using rubcrete instead of concrete at constructions with highly exposed to dynamic loads vibrations was investigated.

## 3. METHODS

The problem discussed in this research consists of three solutions. The first two are theoretical analyses which depend on deriving new equations for simulating the deflection of concrete continuous deep beams under sinusoidal loads. These two methods relenting on the dynamic general equation as well as the Timoshenko beam analysis. While the third solution involved a simulation of concrete beam by ANSYS APDL software and comparing the obtained results with the theoretical results then investigates some case studies to get the full description of parameter effect on the CDBs. Figure 2 shows a block diagram of the complete work and Figure 3 shows the selected CDBs model

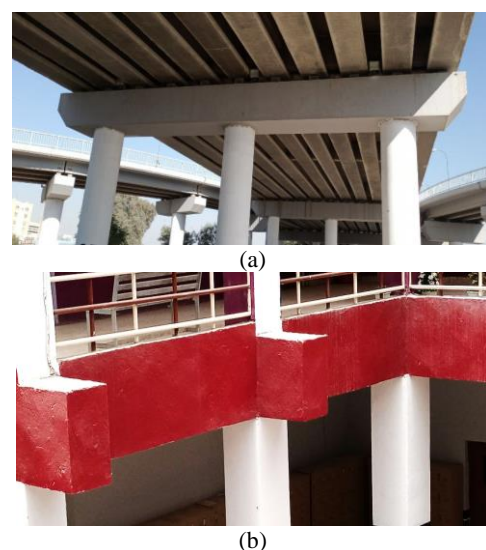


Figure 1. CDBs application at buildings

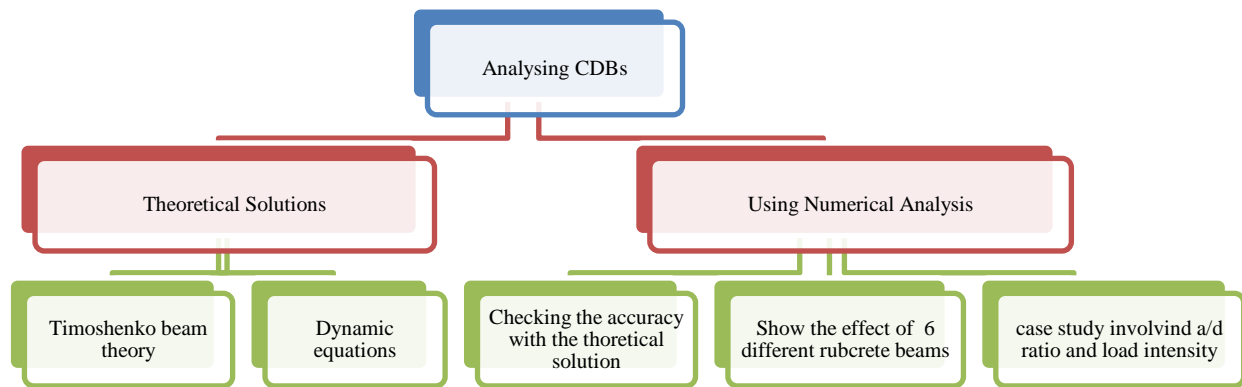


Figure 2. Overall study plane

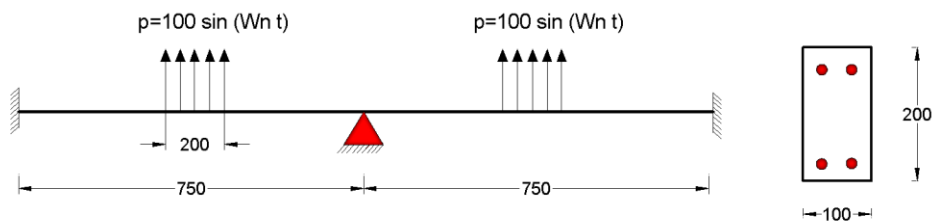


Figure 3. Beam details (all dimensions are in mm)

**3. 1. Timoshenko Beam Solution**

It is difficult to get a plastic theoretical solution for continuous deep beams under the effect of harmonic load. So, an elastic solution has been derived and adopted for verification the analytical solution (by ANSYS V.15.0). Timoshenko beam theory [22] has been used which satisfied the requirement of deep beams, which are:

- The plane section does not remain plane after bending.
- The normal to the neutral plane after bending will not remain normal to the neutral but have an additional rotation due to high transverse shear deformation.
- Neglecting normal strain along the width.

The deriving equation of deep beam deflection is given in Equation (1).

$$\frac{d^4w}{dx^4} = \frac{q}{EI} - \frac{1}{c^2GA} \times \frac{d^2q}{dx^2} \tag{1}$$

Let  $w(m) = \sum_{m=1}^n \sin(\frac{2m\pi x}{l})$  to satisfy the boundary conditions of continuous deep beam.

Substitute in Equation (1), to get the final equation of CDB deflection under harmonic load (Equation (2))

$$w_{at\ mid\ span} = 0.09752 \sin(w_n t) \tag{2}$$

**3. 2. Dynamic Equation Solution**

In general, the structural members resist the dynamic load by its mass, stiffness and the magnitude of damping which depend on the material itself [31]. The damping ratio is ignored in most

structural calculations for safety. For concrete, the damping factor does not exceed 6%. For solving the CDBs under sinusoidal loads, the dynamic equation response of structures is derived with ignoring the effect of deep beams, and as listed below (Equation (3))

$$m\ddot{y} + c\dot{y} + ky = F(t) \tag{3}$$

Applying the same details of beam at Figure 3, to get the time deflection equation for the given beam (Equation (4))

$$y = \frac{F(t)}{\frac{k}{m} - w_n^2} \tag{4}$$

**3. 3. Numerical Simulation**

Beam 188 element has been used for simulating the CDB in ANSYS APDL software [39] (As shown in Figure 4 ). After trying some meshing sizes, it was found that , 20 mm meshing length gives a good agreement with the theoretical model in such fast rendering time. External supports were selected to fixed while the internal was of single vertical reaction.

Seven concrete mixes were simulated depending on Topcu’s tested mixes [40], three percentages used which are (15, 30, 45) % for sand and gravel replacement besides the ordinary conventional mix. All concrete properties were extracted from literature [40]. Compressive strength, tensile strength, concrete unit weight and young’s modulus for all mixes were summarized in Table 1. The input stress versus strain curve is shown in Figure 5. Young’s modulus was

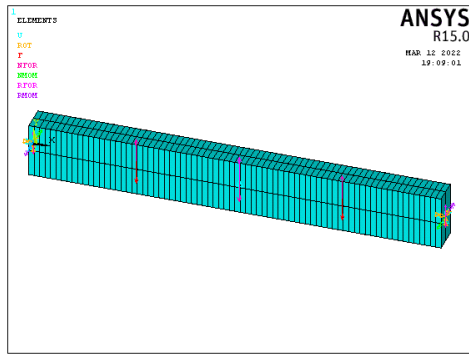


Figure 4. CDB modelling at ANSYS software

TABLE 1. Mechanical concrete properties

Mix	$w_c$ (Kg/m <sup>3</sup> )	$f'_c$ (MPa)	$E_c$ (MPa)
NC	2300	23.48	22983
G15	2220	16.18	18092
G30	2140	12.62	15122
G45	2010	9.9	12192
S15	2220	24.22	22135
S30	2140	19.7	18894
S45	2010	14.77	14892

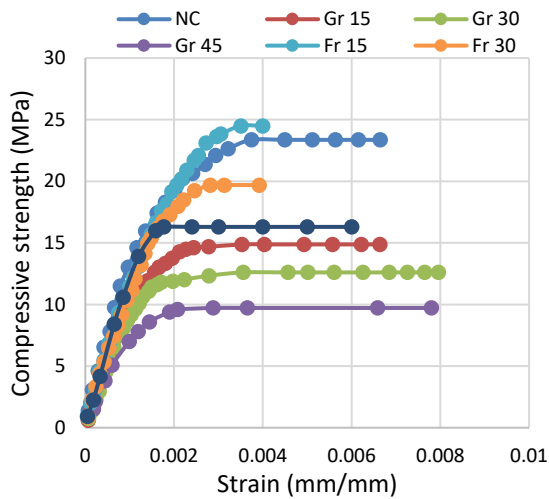


Figure 5. Inserted stress-strain curve into program

considered using ACI 318- 19 (Equation (5)) which is proportional to rubcrete weight (and as adopted for rubcrete mixes reported in literature [41]).

Table 1 shows the details of elastic modulus values for all selected mixes. The analysis was displacement control with a tolerance factor equals 6%.

$$E_c = 0.043 * w_c^{1.5} * \sqrt{f'_c} \quad (5)$$

## 4. RESULTS

### 4. 1. Theoretical Results and Numerical Verification

The validation between numerical and theoretical beam solutions were investigated as illustrated in Figure 6. It can be noted that, a good match in beam response between the numerical and the Timoshenko beam solutions with slight differences in the amplitude between them caused due to that, the nonlinear simulation gives more accurate results. Because it simulates the problem as it is in the nature in a matter accurate than the theoretical solutions. This nonlinear model has been considered to use for simulating a parametric study for rubcrete beams. While, the solution using the general dynamic equation also gives the same response but with less accurate due to ignoring the effect of deep beam effect. It causes an error in deflection magnitudes but the good point, it also gives the same response.

### 4. 2. Rubberized CDBs Results

Nonlinear solution was investigated to study the effect of sinusoidal harmonic load for seven beams. One beam was simulated without any rubber replacement in order to be the reference model, the first group was of three beams with 15, 30, 45% of gravel replacement (GR) and the second group was of the same percentages of fine aggregate replacement (FR). The results for the seven beams as illustrated in Figures 6 to 9, the displacement, velocity and acceleration are discussed. From Figure 6, it can be noted that, the beam of normal concrete (NC) deflects in the less magnitude than the other beam due to its high compressive strength and low elasticity. Beam GR45 showed the maximum values of displacement due to the sinusoidal load because of its low compressive strength, elastic property, and large ability to absorb energy. Also, the negative values of deflection were higher than the positive, which is refers to that the beam approaches from cracking, in another word, if all-deflection points were in negative zone, then the beam is fully cracked. For all beams, the time period does not change in contract with the amplitude.

Velocity of beams data are also presented in Figure 7. It can be noted that, the velocity wave formed as distorted wave (the latter is a combination between fundamental and the third harmonic wave). The same arrangement of amplitude appears at both velocity and acceleration responses, as shown in Figures 7 and 8.

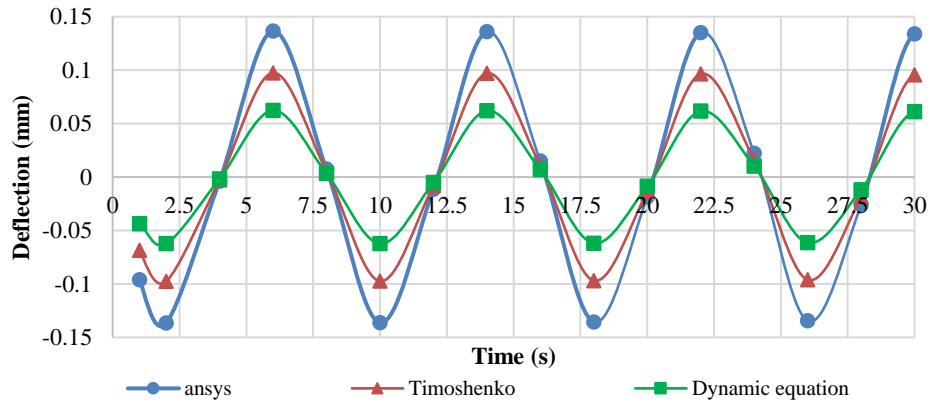
### 4. 3. Case Studies' Results

#### 4. 3. 1. a/h Ratio Influence

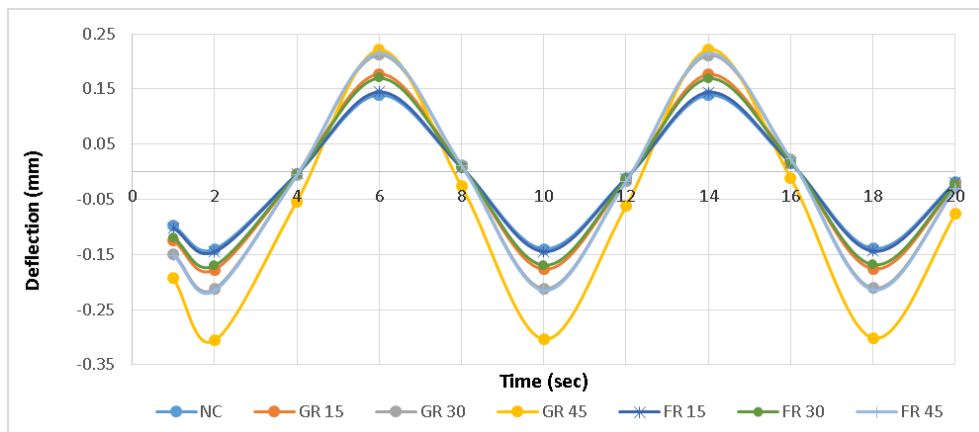
The beam GR 45 showed the weakest model due to its low compressive strength and its high elasticity, so it has been considered for modelling the effect of changing a/h ratio because it represents the more dangerous case of rubcrete. Theoretically, the CDB behavior approaches to be flexural with a smaller a/h ratio, and its shear strength capacity increased linearly with a/h reduction [19]. The smaller a/h

ratio shows minimum deformation capacity of CDB also ductility and beam load capacity [20, 21]. This fact was also noticed in the dynamic analysis of the rubcrete beams. The model of  $a/h=0.5$  showed less displacement against the

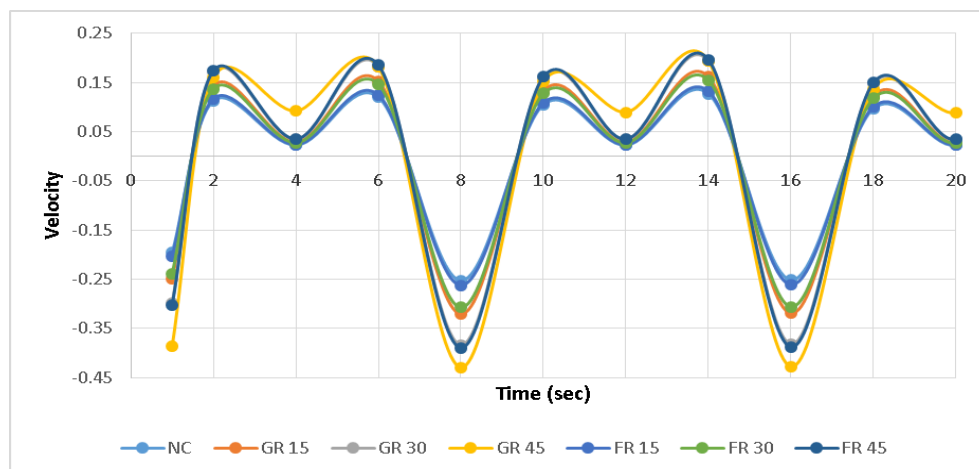
same load and rubcrete properties. The time durations of waves were still deposit but there was a visible difference in beams' amplitudes for all the given deep beams due to  $a/h$  difference, as shown in Figure 9.



**Figure 6.** Verification of deflection by theoretical and analytical solutions



**Figure 7.** Rubcrete CDB deflections



**Figure 8.** Velocity of beams under sinusoidal loading (mm/s)

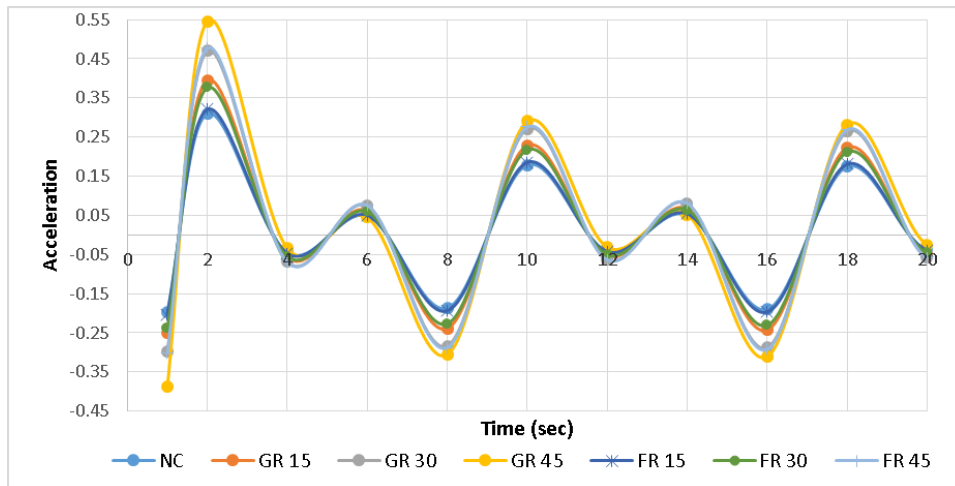


Figure 9. Beams acceleration ( $\text{mm/s}^2$ )

But in case of using a shallow beam ( $a/d=3$ ), the velocity and acceleration amplitudes and periods changed (Figures 10 and 11). The time periods shift to the left with staying on the overall general wave shape. While the deflection (Figure 9) of shallow beam still keeps the periods and the overall general wave shape but with a significant large displacement comparing with the deep beams due to its higher bending capacity.

**4. 3. 2. Load Intensity** The specimen GR45 was selected to study the influence of incrementing load intensity. The original selected load was doubled, tripled, quadruple and quintuple. It can be seen that, the relatively low loads vibrate the beam about the original zero

displacement axis but the higher  $((300 \text{ and } 400) \times \sin W_n t)$  the same wave shape occurred with high negative displacement only. Quintuple load forced the rubcrete model to a plastic flow. Also, from curves in Figure 12, it can be concluded that, the vibration amplitude of rubcrete CDBs depends in the first degree on  $a/h$  ratio, beams' natural frequency and the load intensity. The last conclusion also matched with literature discussions.

The higher displacement occurred when quadrupling the load, the beam deflects more by 80%, and 19.8% comparing with literature.

Figures (13-18) shows the stress versus time for CDBs. It can be noting that, the stress waves gets bigger when increasing harmonic load intensity on beams.

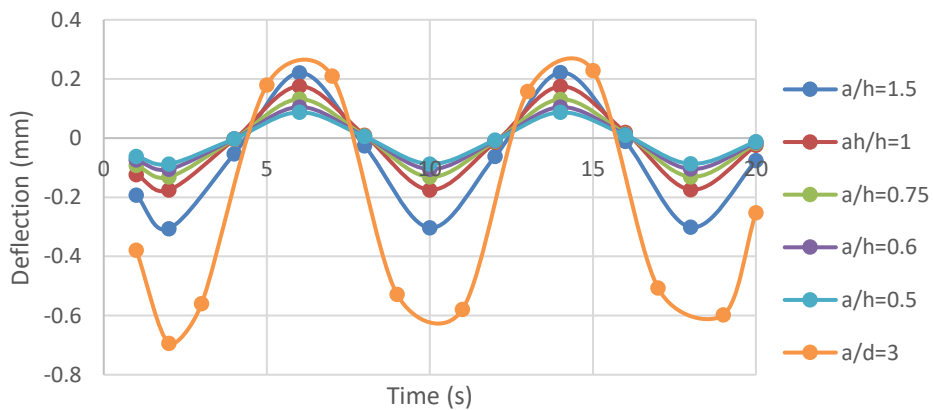
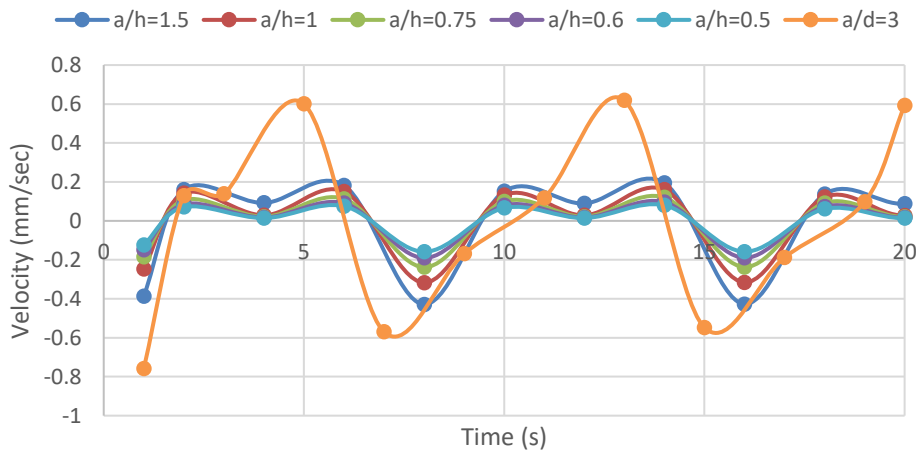
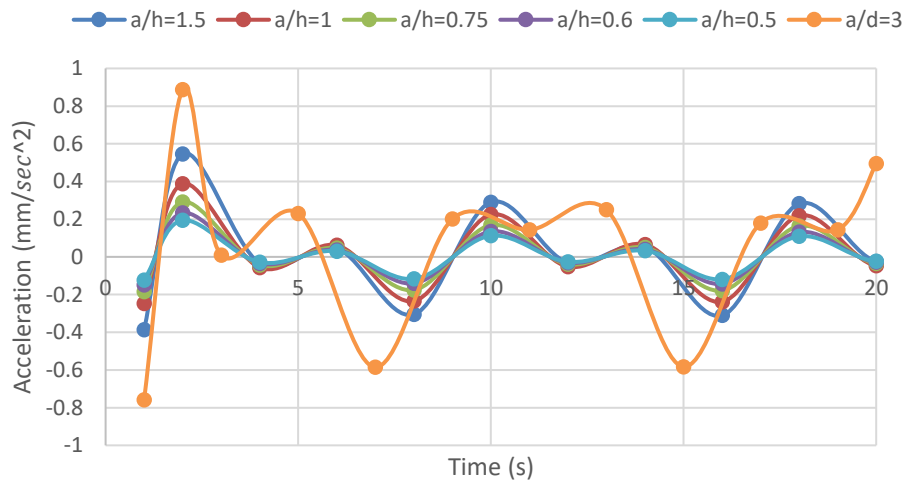


Figure 10. Influence of  $a/h$  ratio on rubcrete beams' deflection

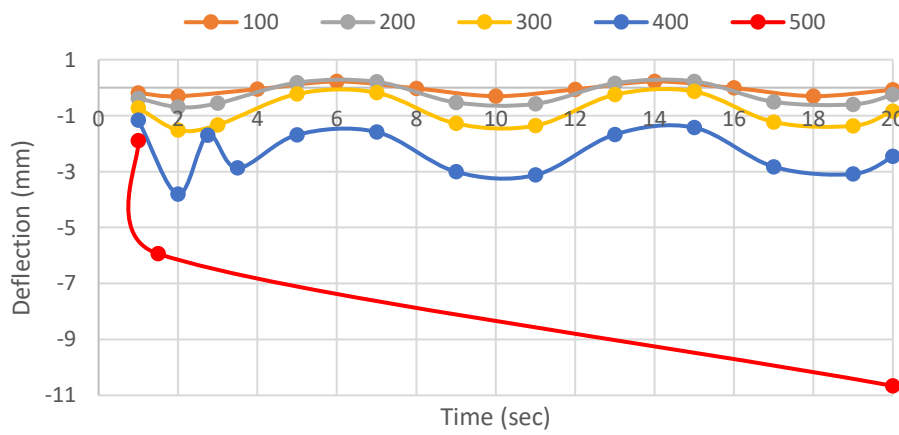




**Figure 11.** Velocity of CDBs of different beams depth



**Figure 12.** Acceleration of CDBs of different depths ((mm/s<sup>2</sup>))



**Figure 13.** Effect of load increment on CDB deflection response

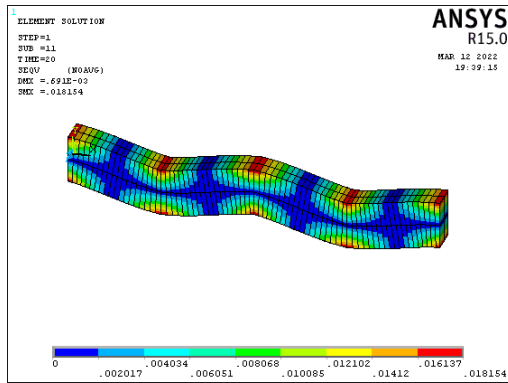


Figure 14. Von-Mises Stress for 100 sin  $W_n t$  loading

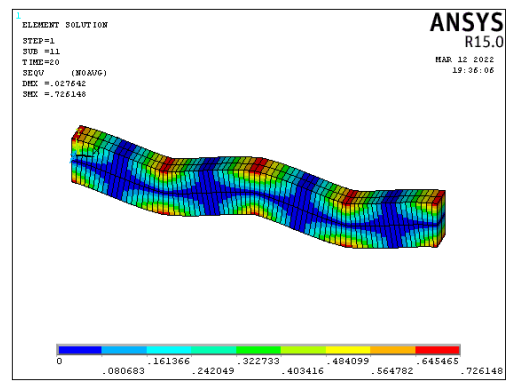


Figure 15. Von-Mises Stress for 200 sin  $W_n t$  case

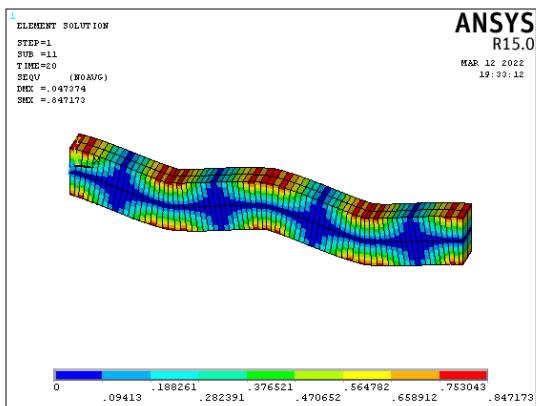


Figure 16. Von-Mises Stress for 300 sin  $W_n t$  case

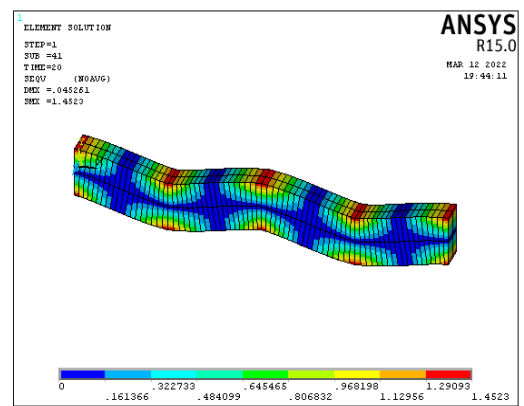


Figure 17. Von-Mises Stress for 400 sin  $W_n t$

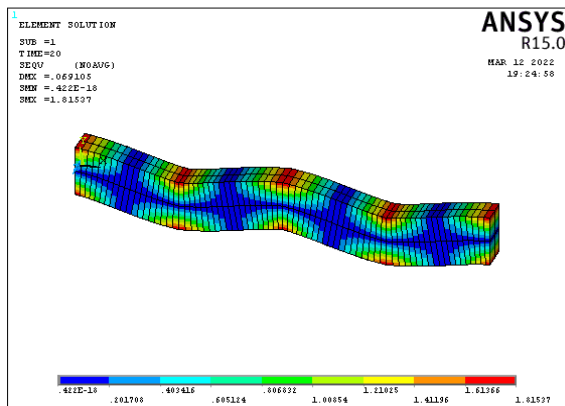


Figure 18. Stress for 500 sin  $W_n t$  loading

5. DISCUSSION

CDBs may be exposed to harmonic loads due to many reasons like vehicle movement on bridges. The solution of CDBs under sinusoidal load and the behaviour of it has not been discovered yet. Timoshenko beam theory was used to discover the behaviour and finding the deflection response along beams. It provided the best match when comparing

with the numerical solution. Concrete is a weak material against energy absorption. So, adding rubber for the mix enhances the dynamic properties. Timoshenko beam theory can be used for harmonic load with a significant degree of exact to the elastic response of CDBs. Also, solving the beam using the general dynamic equations gives a correct estimation for the beam response but an error value of displacement due to ignoring the deep beam effect so it is unfavourite to be used in CDBs analysis. From the numerical analysis, it was found that, Beam element 188 (in spite of its nonlinearity and it is a 2D model) offers a great degree of validation and the exactly same response with the theoretical method. It was found that, the replacement causes a large deflection capacity under the same load. That means the rubberized beam is more flexible against load. Generally, replacing rubber by aggregate leads to an increase in energy absorption and therefore enhanced the beam resistance for dynamic loading.

6. CONCLUSIONS AND FUTURE WORK

From the results, it can be concluded that, using rubber in concrete beams rises the beam deflection and enhances the ductility of beam against dynamic loads. Knowing that,



replacing rubber of a coarse aggregate provides more energy absorption than that of the same percentage of fine aggregate replacement. The sensitivity of concrete deep beams towards a/h ratio is still considerable for harmonic loads, i.e., minimizing the ratio leads to decrementing the deflection wave amplitudes. It was also found that, higher load vibration intensity produces larger displacement amounts, and excessive loads may cause a plastic flow. Furthermore, the vibration amplitude of rubcrete CDBs depends mainly on a/h ratio, beams natural frequency and the load intensity. It is recommended in the future to study the nonlinear theoretical analysis of CDBs under the effect of dynamic loads to find the beam capacity under such conditions.

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### Persian Abstract

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#### چکیده

تیرهای عمیق پیوسته (CDB) پرکاربردترین اعضا در ساخت و سازها هستند و به شدت در معرض انواع بارهای دینامیکی هستند. معروف است که؛ بتن ماده ای شکننده است و مقاومت ضعیفی در برابر جذب انرژی دارد. استفاده از لاستیک تایرهای ضایع شده، جذب انرژی بتن و همچنین برای اهداف پایداری را افزایش می دهد. تئوری تیر تیموشنکو برای حل CDBهای تحت بار سینوسی استفاده شده است و برای تأیید نتایج عددی ANSYS APDL V.15.0 به کار گرفته شده است. هفت مخلوط بتن با نوع و مقادیر متفاوت سنگدانه - جایگزینی لاستیک شبیه سازی شده است. پارامترهای مختلفی مانند نوع جایگزینی، درصد جایگزینی، نسبت برشی تیر به عمق (a/h) و شدت بار مورد مطالعه قرار گرفته است. مشخص شده است که تئوری پرتو تیموشنکو را می توان برای پرتو پیوسته بارگذاری شده هارمونیک به میزان قابل توجهی از پاسخ الاستیک CDBها استفاده کرد. علاوه بر این، جایگزینی به طور کلی به دلیل خاصیت ارتجاعی لاستیک، انعطاف پذیری بیشتری را ارائه می دهد. جایگزینی بتن به میزان ۴۵ درصد، مقادیر جابجایی بیشتری را در میان انواع دیگر دارد. همچنین توجه شده است که حساس بودن تیرهای عمیق بتنی به نسبت (a/h) نسبت به بارهای هارمونیک قابل توجه است، یعنی به حداقل رساندن این نسبت منجر به کاهش دامنه موج انحراف می شود.

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