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Single Machine Scheduling Problem with Batch Outsourcing

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ABSTRACT

Outsourcing as a useful strategy in the industry can be integrated into scheduling problems. Moreover, batch outsourcing is a practical assumption owing to the logistics issues for transferring the parts between the manufacturer and the subcontractors. However, this assumption is rarely addressed in the scheduling literature. In this paper, a novel single machine scheduling problem with the option of batch outsourcing is studied. The objective is to minimize the sum of the total completion time of the jobs and the total outsourcing cost. To solve the problem, first, two mixed-integer linear programming (MILP) models, named MP1 and MP2, are developed, which respectively use a straightforward and an innovative approach to model the outsourcing batches. Next, an optimal property for the outsourcing batches is proven. This property is used to establish a valid inequality for model MP2, which is added to it to obtain a third MILP model, MP3. Extensive computational experiments showed that MP2 outperforms MP1 significantly. Moreover, including the derived valid inequality in MP3 enhances its performance considerably in comparing to MP2. Furthermore, it is observed that MP3 is capable of solving many practical-size problem instances optimally or with a low maximum optimality gap.

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1. INTRODUCTION

Nowadays, companies utilize various strategies to improve their performance and sustain in the competitive market. One of those strategies is to outsource a part of the in-house tasks to relevant partners and subcontractors. This strategy can be used for various reasons in a company such as reducing production costs, compensating capacity constraints, eliminating job overloads, focusing on the main activities of a company, and improving the flexiblity and robustness of the production system [1-3]. Indeed, the outsourcing strategy provides access to a widespread pool of resources out of the company that using them efficiently can significantly increase the competitiveness of the company in the market [4-6].

Outsourcing can be addressed in various fields of operations management such as supply chain management, production planning, logistics, facility layouts, health systems, and etc. Here, some of the related studies in these scopes are reviewed. Accordingly, Aksen et al. [7] studied a facility interdiction problem with demand outsourcing. They modeled the problem as a static Stackelberg game between an attacker and a defender and developed two heuristic algorithms to solve the problem. Alizadeh et al. [8] investigated a capacitated multi-facility location-allocation problem with stochastic demands in which demands of the facilities can also be outsourced to some capacitated sub-sources. The authors proposed a stochastic non-linear mathematical model for the problem to find the optimal location of the facilities and optimal allocation of the demand points to the facilities. Mehdizadeh and Fatehi Kivi [9] addressed a single-item capacitated lot-sizing problem with various assumptions such as setup times, safety stock, backlogging, and inventory capacity. Moreover, in this problem, a part of production at each period can be outsourced to the subcontractors. The authors proposed three metaheuristic algorithms to solve the problem. Parvasi et al. [10] studied a school bus routing problem and bus stops selection in which the decisions are made in two levels, *i.e.* strategical and operational levels. At the strategical level, the location of the stops and routing of the buses are determined, while at the operational level,

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the student demands for using the transportation system are assigned to the buses. Moreover, a part of the student demands can be outsourced as well. In this study, two hybrid metaheuristic algorithms are proposed to solve the defined problem. Giri and Sarker [11] analyzed a supply chain management problem in which the logistics activities are outsourced to a third-party logistics (TPL) service provider. In this problem, there exist a manufacturer with the risk of disruption and multiple independent retailers. The design of the parameters that increases the profitability of the overall supply chain is discussed in this paper. Kim et al. [12] studied a problem related to a global supply chain embracing two divisions: one for production and anotherone for retailing. The production division can be offshored to a low-tax country while for the retail division, outsourcing from an outside supplier is possible. The authors analyzed the optimal choices for the supply chain structure. Heydari et al. [13] studied a supply chain consisting of a manufacturer and a retailer with stochastic demands. A part of production can be outsourced by the manufacturer, and moreover, the order quantity can be updated by the retailer as well. Mathematical models were developed for the studied problem to analyze the effect of outsourcing and the order flexibility possibilities in the problem via numerical experiments. Fathollahi-Fard et al. [14] investigated a home health care supply chain in which the patients' demand can be also outsourced to hospitals directly. A bi-level mathematical programming model representing a static Stackelberg game between nurses and patients is developed for the problem, and efficient metaheuristic algorithms are implemented to solve it.

Scheduling in the factories is a branch of the production management subject that can be integrated into the outsourcing subject as well. Scheduling generally addresses the allocation of resources to tasks over a time horizon [15]. Scheduling problems normally involve various problem constraints and one or more problem objectives to be optimized. In the presence of the outsourcing option in a scheduling problem, the subcontractors are considered as additional resources to perform the jobs over the time. Clearly, involving these extra resources in the problem can enhance the problem objectives and facilitate the problem constraints [16]. Several types of machine environments are addressed by the researchers in the scheduling literature, among which the major types are single machine, parallel machine, flow shop, and job shop [15]. The single machine environment is the simplest scheduling model in which there exists a single machine or server to process a number of jobs. This model occurs in many cases in the real world. Moreover, this model can be considered as a block of more sophisticated systems, particularly being used to analyze the bottlenecks of those systems [17].

Several research works have been conducted in the literature in recent years considering the possibility of outsourcing in a scheduling problem. These research works can be primarily classified with regard to the machine environment type. For comprehensive reviews of this subject based on this classification, we refer the readers to Safarzadeh and Kianfar [16] and Liu [18]. However, the studies related to the context of this paper, *i.e.* the single machine environment or batch outsourcing, are reviewed in the next section in detail.

In the production systems, outsourcing the parts in batches may be a more practical assumption than outsourcing them one by one as the location of subcontractors is normally distant from the in-house plant [19]. Indeed, a considerable amount of time and/or cost is usually consumed for transporting the items between the manufacturer's and the subcontractors' shops, and outsourcing the items in batches helps reduce these logistics times and costs. Furthermore, batch outsourcing may also result in receiving order discounts from the subcontractors and prorate the preparation and setup times and costs required to start the order by them. However, a disadvantage of batch outsourcing is the issue that some parts may be ready for outsourcing while they are required to wait for other parts to be outsourced in batch. Incorporating batch outsourcing in a scheduling problem makes the problem much more complicated as most of the researchers have ignored this practical assumption. In fact, to the best of our knowledge, only four papers in the literature have addressed this assumption, *i.e.* Qi [19, 20, 21], and Ahmadizar and Amiri [22]. It should be noted that the former publication addresses a single machine environment; while, the three others consider a two-stage flow shop scheduling problem. These research works are also reviewed in the next section in detail.

In this paper, a novel single machine scheduling problem is investigated with the option of batch outsourcing. In the considered problem, outsourcing each job involves a certain time and cost for processing it and a logistics time and cost for non-processing issues like packing/unpacking, loading/unloading, and transportation. Meanwhile, all the jobs outsourced in a batch have a common logistics time and cost. The problem objective is to minimize the sum of the total completion time of the jobs and the total outsourcing cost. In order to solve the problem, first, a mixed-integer linear programming (MILP) model, named MP1, is presented using a straightforward modeling approach for the outsourcing batches. Next, a more innovative approach for modeling the outsourcing batches is proposed to develop another MILP model, named MP2. Moreover, an optimal property is proven for the outsourcing batches in the problem, using which a valid inequality for model MP2 is developed. This valid inequality is added to model MP2 to obtain a third MILP model, named MP3. Furthermore, extensive computational experiments are conducted at the end of the paper to evaluate and compare the efficiency of the developed mathematical models.

The remainder of the paper is organized as follows: In section 2, the related literature is reviewed and discussed. In section 3, the considered problem is defined formally, and some real-world connections and applications of it are discussed. In section 4, the proposed solution approaches are presented, and in section 5, numerical experiments are conducted to evaluate the performance of the solution approaches. Finally, the conclusion of the paper is presented in section 6 and possible future studies are discussed.

2. LITERATURE REVIEW

Several research works in the literature have addressed a single machine scheduling problem with the option of outsourcing. Accordingly, Lee and Sung [23] studied a single machine scheduling problem in which each job is either processed in-house or subcontracted. They considered two types of problem objectives for the problem. The first one is to minimize a weighted sum of the total outsourcing cost and the maximum lateness of the jobs, while the second one is to minimize a weighted sum of the total outsourcing cost and the total tardiness of the jobs. In this study, several optimal properties are proven for the problem, based on which a branch and bound and some heuristic algorithms are proposed to solve the problem. Furthermore, the authors have also investigated another similar problem discussed by Lee and Sung [24] with a different problem objective, *i.e.* minimization of a weighted sum of the total outsourcing cost and the total completion time of the jobs. The other assumptions in this study are like the ones of the previous study, and similar solution approaches are also utilized. Qi [19] analyzed a single machine scheduling problem in which the jobs can be outsourced in batch to a single subcontractor. Various problem objectives are discussed in this paper such as makespan, total completion time, number of tardy jobs, and maximum lateness. The objective function of the problem is the sum of one of those problem objectives with the total outsourcing cost. To solve the problem, a dynamic programming approach is applied, and some characteristics of the optimal solutions are proven. Zhong and Huo [25] investigated a single machine scheduling problem in which the completion time of an outsourced job is assumed to be a step function of the job's outsourcing time. The problem is analyzed by two distinct minimization objective functions, *i.e.* the sum of the total cost and the makespan, and the sum of the total cost and the number of delayed jobs. The authors have developed pseudo-polynomial algorithms to solve the problem with the former objective function and a special case of the problem with the latter objective function. Hong and Lee [26] analyzed a single machine scheduling problem in which the jobs can be outsourced to a limited set of subcontractors. The problem objective is to minimize the total outsourcing cost while the jobs' assigned due dates are required to be satisfied. In this paper, several optimal properties are derived for the problem solution, and a pseudo-polynomial algorithm is developed to obtain it. Ren et al. [27] studied a scheduling problem in which there exist a manufacturer with a single machine and a subcontractor with many identical parallel machines. Two problem objectives are considered for the problem, *i.e.* minimization of a weighted sum of the total cost and the total completion time of the jobs and minimization of a weighted sum of the total cost and the makespan. To solve the problem with the former objective, a polynomial-time algorithm is developed; while, for the problem with the latter objective, the problem complexity is analyzed and an approximation algorithm is proposed.

For the possibility of batch outsourcing in a scheduling problem, in addition to Qi [19], which was reviewed before, to our best knowledge, only the three papers being reviewed in the following have addressed this subject. In this regard, Qi [20] studied a flow shop scheduling problem with two serial machines in which the operations corresponding to the first machine can be assigned to a single subcontractor as well. Moreover, all the outsourced items in the scheduling horizon are returned back to the shop in a common batch with a given transportation time. The problem objective is to minimize the makespan and the total outsourcing cost at the same time. The author proposed an algorithm to extract the Pareto optimal solutions and a heuristic approach to estimate them. Furthermore, a two-machine flow shop scheduling problem with batch outsourcing is also studied by Qi [21], in which the operations corresponding to both machines are subject to outsourcing. Accordingly, three outsourcing styles are considered in this paper, *i.e.* two operations of a job are outsourced to a single subcontractor, two operations of a job are outsourced but each one to a different subcontractor, and all the operations for the first machine are outsourced to a certain subcontractor. Meanwhile, the considered problem objective is to minimize the sum of the makespan and the total outsourcing cost. Dynamic programming and other analytical approaches are utilized in this study to analyze the optimal solutions and develop optimal algorithms for the problems. Moreover, Ahmadizar and Amiri [22] considered a two-machine flow shop scheduling problem in which the jobs have different release times. Meanwhile, two certain subcontractors are considered in the problem, each of which corresponds to the operations of one of the in-house machines. As the transportation times and costs are taken into account, the jobs can be transported in batches between the manufacturer and the two subcontractors. The problem objective is to minimize the sum of the makespan and the outsourcing costs. The authors have formulated some mathematical programming models for the problem and also developed an ant colony optimization algorithm (ACO) to solve it.

The papers reviewed above are the major research works we have observed in the literature for the subject of single machine scheduling with outsourcing as well as the subject of scheduling with batch outsourcing. Regarding the correspondence of the reviewed papers to our study, Lee and Sung [23, 24], and Zhong and Huo [25], like our study, have assumed that many subcontractors exist in the problem. However, in their considered problems, each job is outsourced solely, while in our problem, the practical assumption of batch outsourcing is addressed. Furthermore, among the reviewed research works considering batch outsourcing, only Qi [19] addressed the single machine environment. Nonetheless, he has considered a single subcontractor in the problem, and furthermore, assumed that the processing time and cost of an outsourced job are proportional to the in-house processing time of the job via fixed multipliers. However, in the problem defined in our study, the number of subcontractors are not limited like many other research works of scheduling with outsourcing *e.g.* Chen and Li, [28]; Guo and Lei, [29]; Izadi et al., [30], and the outsourcing time and cost of the jobs are arbitrary values, which are independent of the in-house processing times.

3. PROBLEM DESCRIPTION

In the considered problem, n jobs are available in the manufacturer's shop, which can be processed by a single in-house machine. Moreover, each job can be also outsourced to the existing subcontractors instead of being processed in the shop. It is supposed that enough number of subcontractors exist in the problem such that the jobs can be outsourced in parallel with no restriction. If job i is fulfilled in the in-house shop, then it has a processing time of p_i, and if it is outsourced, it has a processing time of q_i and a processing cost of c_i. Furthermore, it is assumed that outsourcing a job encompasses a *logistics time*, α_1, and a *logistics cost*, α_2, which are for the activities like packing/unpacking, loading/unloading, and transportation. Moreover, the logistics time and cost can also involve the preparation time and cost required by the subcontractor to accept and start the order. In addition, it is possible to outsource a set of jobs to a subcontractor in batch with the aim of reducing the logistics outsourcing times and costs. In this case, a common logistics time and cost are considered for the jobs included in an outsourcing batch. Moreover, it is supposed that the jobs in a batch are returned back to the shop together, *i.e.* they have identical completion times. This assumption is useful for the cases that the orders are delivered to the customers from the in-house shop or the parts require additional production processes in the in-house shop. To have a more precise expression, suppose that jobs $i_1. i_2. \cdots . i_k$ are outsourced in a batch to a subcontractor. Then, the completion time of each job in the batch is equal to $\alpha_1 + \sum_{j=1}^{k} q_{i_j}$. Moreover, the total outsourcing cost of the jobs in the batch is equal to $\alpha_2 + \sum_{j=1}^{k} c_{i_j}$. The considered objective for the problem is to minimize the sum of the total completion time of the jobs and the total outsourcing cost. Note that the time and cost parameters are normalized before, to make aggregating the mentioned terms in the objective function rational.

A real application for the above-defined problem is the case in which a manufacturer can outsource machining of some of its raw materials to external CNC shops. This strategy is used for the times that the internal jobs are overloaded or some in-house resources are disrupted. CNC shops in some industrial areas are frequently located. For example, in the west region of Tehran, there exist many shops that do machining processes such as turning, milling, etc. They also normally have similar processing times and fees. Moreover, it is supposed that the manufacturer is distant from the location of the CNC shops, for example, it is located in the Shamsabad industrial city in the south of Tehran province which normally encompasses medium and large factories. So, transferring the parts between the manufacturer's shop and the CNC shops requires a considerable amount of logistics times and costs, which makes using the batch outsourcing strategy indispensable.

Finally, the utilized notations for the problem parameters are summarized as follows:

n : number of jobs.
p_i : in-house processing time of job i.
q_i : outsourcing processing time of job i.
c_i : outsourcing processing cost of job i.
α_1 : logistics time for an outsourcing batch.
α_2 : logistics cost for an outsourcing batch.

4. SOLUTION APPROACHES

Mathematical programming is used extensively to formulate combinatorial optimization problems, and especially, scheduling problems [31]. Although most of the scheduling problems are NP-hard, many of them, at least for a considerable segment of practical sizes, can be solved optimally or with a low optimality gap by appropriately developed MILP models. Furthermore, the efficiency of the mathematical programming approach for solving optimization problems is increasing nowadays due to the enhancement of solution techniques and development of computer technologies. This fact has made the mathematical programming solution approach more attractive for researchers and practitioners in recent years [32,33].

In this section, three MILP models are proposed to solve the studied problem. The first MILP model is developed using a straightforward modeling approach, while a more advanced modeling approach is utilized to establish the second MILP model. Moreover, the third MILP model is an enhancement of the second MILP model based on an optimal property of the problem being proven while expressing the model. In the following, first, a simple property for the optimal solution of the problem is proven in Lemma 1, which is used in developing all the aforementioned MILP models.

Lemma 1. In the optimal solution of the problem, the jobs in the in-house shop are processed by the shortest processing time rule.

Proof. Suppose that J_1 and J_2 are two in-house processed jobs in the optimal solution of the problem having respectively processing times of p and q. Moreover, suppose that n_1 jobs are processed after J_1, and n_2 jobs are processed after J_2 in the in-house shop so that $n_1 > n_2$, *i.e.* J_1 is processed before J_2. If these two jobs are replaced with each other in the in-house sequence of jobs, then the amount added to the value of the objective function is equal to $(n_1+1)q+(n_2+1)p-(n_1+1)p-(n_2+1)q$, which is simplified as $(n_1-n_2)(q-p)$. However, according to the optimality of the solution, it holds that $q \geq p$, which concludes the proof.

In order to use the result of Lemma 1 in the MILP models, let suppose that $l_1, l_2, \dots, l_n$ is a sequence of job numbers in which $p_{l_i} \geq p_{l_j}$ for $i < j$. This sequence gives the order of jobs with regard to the processing time, which will be used later in establishing some constraints of the proposed MILP models.

4. 1. Model MP1 The major difference between the first and the second developed MILP models is in the way of modeling the outsourcing batches. As was mentioned before, the first MILP model, MP1, uses a straightforward approach to model the outsourcing batches. In this respect, a distinct index is considered to determine the outsourcing batches in the model. As the number of outsourcing batches in a problem solution is at most n, the considered domain for the aforementioned index is 1 to n. However, many of the n corresponding batches modeled in this way may be empty in the solution of the problem. The details of the model MP1 are presented as follows:

Decision variables:

b_j: binary variable, equals to 1 if outsourcing batch j is non-empty; 0 otherwise.

z_{ij}: binary variable, equals to 1 if job i is included in outsourcing batch j; 0 otherwise.

T_i: real variable, represents the completion time of job i.

Model:

$$\text{Min } \sum_{i=1}^{n} T_i + \alpha_2 \sum_{j=1}^{n} b_j + \sum_{i=1}^{n}\left(c_i \sum_{j=1}^{n} z_{ij}\right) \quad (1)$$

Subject to:

$$\sum_{j=1}^{n} z_{ij} \leq 1 \qquad 1 \leq i \leq n \quad (2)$$

$$\frac{\sum_{i=1}^{n} z_{ij}}{n} \leq b_j \quad 1 \leq j \leq n \quad (3)$$

$$T_i \geq -M_1\left(1-z_{ij}\right) + \alpha_1 + \sum_{k=1} q_k z_{kj} \quad 1 \leq i \,.\, j \leq n \quad (4)$$

$$T_{l_i} \geq -M_2\left(\sum_{j=1}^{n} z_{l_i j}\right) + p_{l_i} + \sum_{k=i+1}^{n} p_{l_k}\left(1-\sum_{j=1}^{n} z_{l_k j}\right) \qquad 1 \leq i \leq n \quad (5)$$

In the above model, Expression 1 describes the objective function of the problem. Constraint 2 guarantees that each job is included at most in one outsourcing batch. Constraint 3 indicates that if outsourcing batch j is not empty, then its corresponding decision variable, b_j, is equal to 1. Constraint 4 determines the completion times of the outsourced jobs, while constraint 5 specifies the completion times of the in-house processed jobs. Note that in these two constraints, M_1 and M_2 are big enough numbers that make the constraints inactive in the intended cases. In this respect, constraint 4 is trivial if job i is not included in batch j, while constraint 5 has no effect in the model if job i is outsourced. It should be noted that the minimum appropriate values for M_1 and M_2 are $\alpha_1 + \sum_{k=1}^{n} q_k$ and $\sum_{k=1}^{n} p_k$, respectively. Moreover, note that the result given in Lemma 1 is utilized in formulating constraint 5 using notations given after Lemma 1.

4. 2. Model MP2 In the second proposed MILP model, MP2, a more innovative approach is utilized to model the outsourcing batches with the hope of establishing a more efficient model. It is worth noting that in model MP1, the maximum possible number of the batches were taken into account, while many of them may be empty in the obtained solution. This fact may cause increasing the dimensions of the model and wasting the computational time consumed for searching the solution space. However, in model MP2, unlike model MP1, the outsourcing batches are not directly distinguished from each other by an index. Accordingly, a binary decision variable like b_j, called the *batch indicator variable*, is utilized instead in the model to enumerate the number of outsourcing batches. This variable is equal to 1 if job j is outsourced and has the minimum job number in its outsourcing batch. In this way, for each existing outsourcing batch, one and only one of the batch indicators variables will be equal to 1. So, the number of outsourcing batches can be obtained in the model via this variable. Moreover, another binary decision variable like v_{ij}, called the *batching variable*, is also defined, which is equal to 1 if jobs i and j are outsourced in a common batch, and 0 otherwise. The details of the model MP2 are given in the following:

Decision variables:

b_i : binary variable, equals to 1 if job i is outsourced and has the minimum job number in its outsourcing batch; 0 otherwise.

v_{ij} : binary variable ($i < j$), equals to 1 if jobs i and j are outsourced in a common batch; 0 otherwise.

f_i : binary variable, equals to 1 if job i is outsourced; 0 otherwise.

T_i : real variable, represents the completion time of job i.

Model:

$$\text{Min } \sum_{i=1}^{n} T_i + \alpha_2 \sum_{i=1}^{n} b_i + \sum_{i=1}^{n} c_i f_i \tag{6}$$

Subject to:

$$2\, v_{ij} \leq f_i + f_j \quad 1 \leq i < j \leq n \tag{7}$$

$$v_{ij} + v_{jk} - v_{ik} \leq 1 \quad 1 \leq i < j < k \leq n \tag{8}$$

$$f_i \leq b_i + \sum_{k=1}^{i-1} v_{ki} \quad 1 \leq i \leq n \tag{9}$$

$$T_i \geq -M_1(1 - f_i) + \alpha_1 + q_i + \sum_{k=1}^{i-1} q_k v_{ki} + \sum_{k=i+1}^{n} q_k v_{ik} \quad 1 \leq i \leq n \tag{10}$$

$$T_{l_i} \geq -M_2 f_{l_i} + \sum_{k=i+1}^{n} p_{l_k}(1 - f_{l_k}) \quad 1 \leq i \leq n \tag{11}$$

The objective function of the problem is presented in Expression 6. Moreover, constraint 7 ensures that if two jobs are included in the same outsourcing batch, their relevant outsourcing indicator variables are equal to 1. Constraint 8 states a *transitive* relationship between the decision variables v_{ij}, *i.e.* if jobs i and j present in the same outsourcing batch as well as jobs j and k, then jobs i and k are also in a common outsourcing batch. Constraint 9 states that if job i is outsourced, and no job exists in its outsourcing batch with a lower job number than i, then b_i is equal to 1, *i.e.* the batch is identified by job i in the model. Finally, Constraints 10 and 11 determine the completion times of the outsourced jobs and the in-house processed jobs, respectively. Meanwhile, the minimum appropriate values for M_1 and M_2 in this model are $\alpha_1 + \sum_{k=1}^{n} q_k$ and $\sum_{k=1}^{n} p_k$, respectively.

4. 3. Model MP3 Although the computational results in the next section indicate that model MP2 significantly outperforms model MP1 in solving numerical problem instances, here, we modify MP2 to generate another model, named MP3, with the hope of having even a more efficient model. In this regard, first, a notable optimal property for the outsourcing batches in the problem is proven in Lemma 2. Then, this property is used to develop a valid inequality for model MP2, which is added to the model as a new constraint to obtain the model MP3. In fact, this constraint helps to reduce the solution space of the MILP model, which may enhance its efficiency in searching for the optimal solution.

Lemma 2. In the optimal solution of the problem, the outsourcing batches can be represented by a sequence like $B_1, B_2, \ldots, B_k$ such that if $i < j$, then each job in B_i has an equal or bigger outsourcing time than each job in B_j.

Proof. Suppose that in the optimal solution of the problem, B_{i_1} and B_{i_2} are two arbitrary outsourcing batches containing respectively n_1 and n_2 jobs, such that $n_1 \leq n_2$. Moreover, job J_1 with an outsourcing time of p and job J_2 with an outsourcing time of q are included in batches B_{i_1} and B_{i_2}, respectively. Now, suppose that J_1 and J_2 are replaced with each other. Then, it can be easily checked that the amount of $n_1(q - p) + n_2(p - q)$, which is simplified as $(n_1 - n_2)(q - p)$, is added to the value of the objective function. However, this term must be non-negative according to the optimality of the solution. So, it is concluded that $p \geq q$ if $n_1 < n_2$. Moreover, if $n_1 = n_2$, replacing J_1 and J_2 with each other has no effect on the value of the objective function, so without loss of generality, we assume $p \geq q$ for this case. Accordingly, if we sort the outsourcing batches in the optimal solution of the problem in the form of a sequence like $B_1, B_2, \ldots, B_k$ such that for $i < j$, the number of jobs in B_i is smaller than or equal to the number of jobs in B_j, the proof is concluded. □

Now, we establish a valid inequality for model MP2 using the result obtained via Lemma 2. First, let suppose without loss of generality that the jobs are numbered in descending order with regard to the outsourcing processing time parameter, *i.e.* $q_i \geq q_j$, if $i < j$. According to this assumption, the batch indicator jobs in model MP2, for which the variable b_i is equal to 1, are the ones that have the maximum outsourcing time in their containing outsourcing batch since they have minimum job numbers in their batch. Regarding Lemma 2, we know that if a batch indicator variable like b_i in model MP2 is equal to 1, then any job k before job i ($k < i$) and any job l after job i or job i itself ($l \geq i$) cannot be included in an identical outsourcing batch. In fact, if jobs k and l are in a common outsourcing batch, and job i is also an outsourced job, then, according to Lemma 2, jobs k, i, and l are definitely in the same outsourcing batch, which is a contradiction with the assumption $b_i = 1$. Regarding this result, we establish the following valid inequality for model MP2:

$$\sum_{k=1}^{i-1} \sum_{l=i}^{n} v_{kl} \leq (i-1)(n-i+1)(1-b_i) \quad 1 \leq i \leq n \tag{12}$$

In constraint 12, if the batch indicator variable b_i is equal to zero, the constraint will be trivial as the left-hand side of it is at most $(i-1)(n-i+1)$ in any case. However, if b_i is equal to one, the abovementioned result for jobs k, i, and l will be expressed by constraint 12. Hence, constraint 12 is always true for model MP2. As was discussed before, the MILP model MP3 is simply obtained by adding constraint 12 to model MP2. In the next section, the usefulness of this modification is demonstrated via numerical results.

5. COMPUTATIONAL EXPERIMENTS

In this section, the performance of the proposed MILP models in the previous section are examined through numerical experiments. In this respect, some appropriate numerical instances of the problem are prepared first as the test problems. Next, the problem instances are solved by the MILP models MP1, MP2, and MP3 via CPLEX software. Finally, the experimental results are presented and discussed, and the performance of the models are evaluated and compared. The models are solved by CPLEX 12.6.0 and implemented in Visual Studio 2017 via the Concert Technology of CPLEX software. The computer systems used to run the models have characteristics of CPU 2.4 GHz, 4 GB RAM, and OS of Windows 8.

5. 1. Test Problems In order to conduct the computational experiments, first, it is required to provide some adequate test problems. In this respect, the problem dimensions n =10, 15, 20, 25, 30, 40, and 50 are chosen to generate the test problems. Moreover, it is required to generate the values of the parameters p_{ij}, q_{ij}, c_{ij}, α_1 and α_2 for the considered problem dimensions. Accordingly, for each problem dimension, these numerical values are generated from three different sets of discrete uniform probability distributions, named S_1, S_2, and S_3, which are presented in Table 1. In this table, Uni[a b] represents a discrete uniform probability distribution with a lower bound a and an upper bound b. Note that the considered uniform probability sets are chosen using preliminary tests such that various levels of outsourcing are obtained in the test problems. According to the considered distributions, the problem instances related to set S_2 have the lowest, and the problem instances related to set S_3 have the highest levels of outsourcing.

It should be noted that for each problem dimension with a set of probability distributions, 10 problem instances are generated randomly, which are named a *problem class* as a whole. Consequently, 210 problem instances organized in 21 problem classes are prepared to be used in the experiments.

5. 2. Experiments and Results To conduct the numerical experiments, the developed MILP models MP1, MP2, and MP3 are solved by CPLEX software for each of the 210 problem instances generated in section 5.1. It should be noted that the values of the parameters M_1 and M_2 in the MILP models are set to the minimum values mentioned while expressing the models in section 4. Moreover, a time limit of one hour is considered for running each of the models for a problem instance. Note that if a problem instance is not solved optimally within the assigned time limit, an optimality gap will be reported by CPLEX, which is equal to $\frac{objective\ value - lower\ bound}{objective\ value} \times 100$ in which the *objective value* is the best obtained value for the objective function, and the *lower bound* is the best obtained lower bound by the software. The optimality gap indicates the maximum relative deviation of the attained solution from the optimal solution of the problem. Accordingly, if this value is low, we can be sure that the model has attained a near-optimal or even an optimal solution.

The experimental results obtained by running the MILP models are given in Table 2. In this table, each row corresponds to the results of the problem instances of a problem class. Accordingly, the indicator ST represents the average solution time for the corresponding problem instances. Meanwhile, all the time indicators are reported in seconds in Table 2. Note that if a MILP model is not solved within the one-hour time limit, its solution time will be equal to 3600 seconds. Furthermore, the indicator OF represents the average value of the objective function obtained by the MILP model among the problem instances. Moreover, the indicator $G\%$ gives the average value of the optimality gap reported by CPLEX for the problem instances. Obviously, if a problem instance is solved optimally within the assigned time limit, the optimality gap will be zero. Furthermore, the number of problem instances for the corresponding problem class that are solved optimally by the MILP model is reported by the indicator $opt\#$, which is out of 10. Finally, the average percentage of the outsourced jobs in the obtained problem solutions is presented by the indicator $out\%$.

TABLE 1. The sets of probability distributions used to generate the test problems

	p_{ij}	q_{ij}	c_{ij}	α_1	α_2
S_1	Uni[10 100]	Uni[10 100]	Uni[10 100]	Uni[50 250]	Uni[50 250]
S_2	Uni[10 50]	Uni[10 100]	Uni[10 200]	Uni[50 150]	Uni[50 300]
S_3	Uni[10 200]	Uni[10 50]	Uni[10 100]	Uni[50 250]	Uni[50 150]

TABLE 2. The detailed results of the numerical experiments

Problem Class		Model MP1				Model MP2				Model MP3				
n	type	OF	ST	G%	$opt\#$	OF	ST	G%	$opt\#$	OF	ST	G%	$opt\#$	out%
10	S_1	2346	0.53	0	10	2346	0.19	0	10	2346	0.2	0	10	27
10	S_2	1521	0.06	0	10	1521	0.04	0	10	1521	0.05	0	10	11
10	S_3	2664	1.92	0	10	2664	0.27	0	10	2664	0.23	0	10	53
15	S_1	3297	92	0	10	3297	4.41	0	10	3297	3.18	0	10	43
15	S_2	2654	1.81	0	10	2654	0.68	0	10	2654	0.61	0	10	22
15	S_3	3363	1295	8.2	5	3363	37	0	10	3363	2.81	0	10	65
20	S_1	5413	3600	24.28	0	5407	2656	3.75	4	5407	735	0.53	8	50
20	S_2	4408	907	2.58	8	4408	335	0	10	4408	97	0	10	29
20	S_3	5069	3600	32	0	5069	3283	5.35	1	5069	250	0	10	70
25	S_1	6815	3600	40.53	0	6809	3600	6.7	0	6805	3020	3.15	4	58
25	S_2	5875	3600	30.95	0	5873	3600	5.93	0	5872	2731	3.1	6	39
25	S_3	6331	3600	67.78	0	6314	3600	6.9	0	6314	3122	4.75	3	73
30	S_1	7496	3600	49.6	0	7489	3442	6.85	1	7488	3343	4.21	2	59
30	S_2	6828	3600	36.47	0	6827	3600	8.37	0	6824	3270	4.34	2	44
30	S_3	7344	3600	70.25	0	7341	3600	8.03	0	7341	3517	5.78	1	74
40	S_1	11772	3600	64.21	0	11720	3600	19.2	0	11697	3600	16.11	0	65
40	S_2	10465	3600	55.88	0	10447	3600	19.83	0	10428	3600	17.08	0	54
40	S_3	11137	3600	67.62	0	11107	3600	14.3	0	11093	3600	11.26	0	74
50	S_1	15437	3600	66.62	0	15315	3600	23.87	0	15302	3600	21.63	0	67
50	S_2	14487	3600	57.8	0	14436	3600	27.54	0	14413	3600	23.22	0	56
50	S_3	14105	3600	69.18	0	14007	3600	20.75	0	13995	3600	18.72	0	77

This quantity shows the average level of job outsourcing in each problem class. Note that the values presented in Table 2 are rounded in some cases to have a better representation of the indicators in the table.

In addition to the results reported in Table 2, the average values of the indicators ST, OF, and G% for each problem dimension are also given in Table 3. Therefore, each quantity in Table 3 is the average of the three related quantities in Table 2 for the corresponding problem dimension. Using the results of Table 3, the performance of the MILP models can be compared quickly. Moreover, to have an intuitive comparison of the efficiency of the MILP models, the values of the average solution time, ST, and the average optimality gap, G%, reported in Table 3 are also illustrated in Figures 1 and 2, respectively.

The experimental results indicate that model MP2 outperforms model MP1, and model MP3 outperforms model MP2 in solving the numerical problem instances. Particularly, the performance of models MP2 and MP3 in proving solution optimality or estimating the maximum optimality gap is significantly better than the performance of model MP1. Moreover, by considering the results for the problem dimensions that the problem instances are generally solved optimally for, it is observed that the solution times of models MP2 and MP3 has been significantly lower than the solution times of model MP1. The aforementioned results are evident

TABLE 3. The average values of the experiment indicators for the problem dimensions

	Model MP1			Model MP2			Model MP3		
n	OF	ST	G%	OF	ST	G%	OF	ST	G%
10	2177	0.84	0	2177	0.17	0	2177	0.16	0
15	3105	462	2.73	3105	14	0	3105	2.2	0
20	4963	2702	19.62	4961	2091	3.03	4961	360	0.18
25	6340	3600	46.42	6332	3600	6.51	6330	2957	3.67
30	7223	3600	52.11	7219	3547	7.75	7218	3376	4.78
40	11125	3600	62.57	11091	3600	17.78	11073	3600	14.82
50	14676	3600	64.53	14586	3600	24.05	14570	3600	21.19

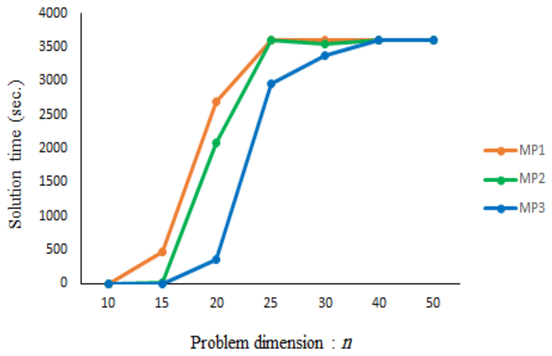

Figure 1. The average solution times for the problem dimensions

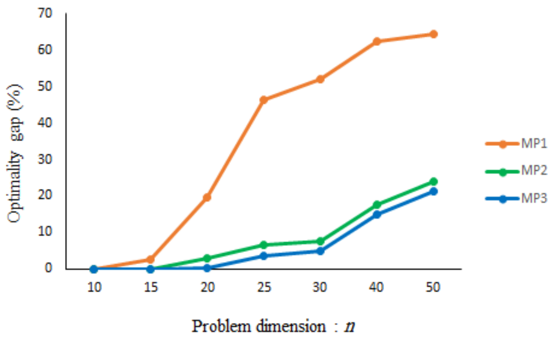

Figure 2. The average optimality gaps for the problem dimensions

in Table 3, Figures 1 and 2. So, we can conclude that the innovative modeling approach for the outsourcing batches utilized in MP2 and MP3 has been an effective approach. Moreover, regarding the results in Table 3, Figures 1, and 2, it is observed that the average optimality gaps obtained by MP3 have been lower than the ones of MP2, and the average solution times of MP3 have been also lower than the average solution times of MP2. So, the valid inequality obtained by the optimal property proven in Lemma 2, which is included in model MP3 has been also successful in improving the performance of the model.

Furthermore, regarding the results of Table 2, it is observed that using model MP3 as the most efficient model, almost all the problem instances with up to 20 jobs are solved optimally within the assigned time limit. Moreover, the problem instances with 25 and 30 jobs are also solved with a maximum optimality gap below 5% on average. Furthermore, this quantity is almost below 20% on average for the instances with 40 and 50 jobs. It is worth noting that the mentioned values are the maximum possible optimality gaps, and the real optimality gaps for the obtained solutions may be much lower. According to the abovementioned results, we can generally conclude that model MP3 is an efficient model to solve practical-size problem instances of the studied problem.

Moreover, another notable result regarding the last column of Table 2 is that a considerable segment of the jobs, around 40 to 50 percent of them on average, are outsourced in the obtained solutions. This result indicates the great role that the outsourcing strategy can have in the addressed problem. However, the amount of outsourcing in the problem is evidently related to the values of the outsourcing time and cost parameters. In this regard, it is observed that when the outsourcing parameters are generated from probability distribution sets S_2, the rate of outsourcing is minimum, and when they are generated from probability distribution sets S_3, the rate of outsourcing is maximum. So, the results also indicate that the impact of the outsourcing strategy is significantly related to the combination of values of the outsourcing time and cost parameters.

6. CONCLUSION

In this paper, a novel single machine scheduling problem with the option of batch outsourcing was studied. The considered problem objective was to minimize the sum of the total completion time of the jobs and the total outsourcing cost. To solve the problem, three MILP models were developed. The first one, called MP1, exploited a straightforward modeling approach for the outsourcing batches, while, a more advanced approach for modeling outsourcing batches was utilized in the next MILP model, MP2. Furthermore, to enhance the efficiency of MP2 more, an optimal property for outsourcing batches was proved, which was used to develop a valid inequality for MP2. By adding this inequality to MP2 as a new constraint, another MILP model, named MP3, was also obtained.

At the end of the paper, extensive numerical experiments were conducted to evaluate and compare the efficiency of the MILP models. The results indicated that MP2 outperforms MP1, and MP3 outperforms MP2 in solving the numerical problem instances. Particularly, the results obtained by MP2 and MP3 were significantly better than the ones of MP1 with regard to the solution time and optimality reporting. Moreover, the numerical results indicated that the valid inequality included in MP3 has been effective in improving the performance of the MILP model. Furthermore, it was concluded that model MP3 can generally solve medium-size problem instances optimally or with a low optimality gap in a reasonable time. In addition, as a managerial insight, it was observed that the outsourcing strategy can have a great impact in the defined problem since a considerable segment of the jobs were outsourced in the solutions of the numerical problem instances. However, the rate of outsourcing is dependent on the values of the outsourcing time and cost parameters.

For future studies, we propose enhancing the solution approaches given in this paper for the studied problem or utilizing other solution techniques such as heuristic and meta-heuristic algorithms. Moreover, other problem objectives, especially due-date related ones, can be also considered for the studied problem. Furthermore, the practical assumption of batch outsourcing, which is rarely addressed in the literature, can also be investigated for scheduling problems in other machine environments such as parallel machine, flow shop, and job shop.

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Persian Abstract

چکیده

برون سپاری یک راهبرد متداول در صنعت است که می تواند به عنوان یک فرض در مسائل زمان بندی نیز در نظر گرفته شود. علاوه بر این، برون سپاری دسته ای کارها به دلیل وجود مسائل لجستیکی انتقال قطعات بین تولید کننده و پیمانکاران یک موضوع کاربردی است که در ادبیات مسائل زمان بندی به ندرت بررسی شده است. در این تحقیق، یک مساله جدید زمان بندی تک ماشین با امکان برون سپاری دسته ای کارها و با تابع هدف کمینه سازی مجموع زمان تکمیل کارها و هزینه های برون سپاری بررسی می شود. برای حل مساله ابتدا دو مدل برنامه ریزی خطی عدد صحیح آمیخته با نام های MP1 و MP2 ارائه می شود که به ترتیب از یک رویکرد سر راست و یک رویکرد ابتکاری جهت مدل سازی دسته های برون سپاری بهره می گیرند. سپس یک ویژگی بهینه برای دسته های برون سپاری به صورت دقیق به اثبات می رسد که با استفاده از آن یک نامساوی معتبر برای مدل MP2 توسعه داده می شود. با افزودن این نامساوی به عنوان یک محدودیت به مدل MP2، یک مدل سوم برنامه ریزی عدد صحیح آمیخته با نام MP3 نیز بدست می آید. در انتهای مقاله آزمایش های محاسباتی جهت ارزیابی عملکرد مدل های پیشنهادی انجام می شود که نتایج بدست آمده نشان دهنده برتری قابل توجه کارایی مدل MP2 نسبت به مدل MP1 و مدل MP3 نسبت به مدل MP2 است. همچنین مشاهده می شود که با استفاده از مدل MP3، تعداد زیادی از مسائل با ابعاد کاربردی به صورت بهینه و یا با درصد خطای بهینگی کم قابل حل است.