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A Robust Knapsack Based Constrained Portfolio Optimization

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ABSTRACT

Many portfolio optimization problems deal with allocation of assets which carry a relatively high market price. Therefore, it is necessary to determine the integer value of assets when we deal with portfolio optimization. In addition, one of the main concerns with most portfolio optimization is associated with the type of constraints considered in different models. In many cases, the resulted problem formulations do not yield in practical solutions. Therefore, it is necessary to apply some managerial decisions in order to make the results more practical. This paper presents a portfolio optimization based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints for asset allocation. To handle the uncertainty associated with different parameters of the proposed model, we use robust optimization techniques. The model is also applied using some realistic data from US stock market. Genetic algorithm is also provided to solve the problem for some instances.

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1. INTRODUCTION

Markowitz [1, 2] theorem presents a portfolio selection model with continuous variables that simultaneously considers the risk and returns of the portfolio. One of the primary concerns with the classical mean-variance model based on the continuous variables is that the optimal result may not be necessarily applicable for asset allocation. In the past, the managers of most companies used to be interested in splitting the shares once in a while. However, presently, there is an increase in the number of firms whose managers are reluctant to split the shares of their companies; for instance, AMAZON, GOOGLE, etc. Therefore, in order to trade shares, the only possible way that remains is in the matter of integer values of them. Hence, the rounding problem is getting more important than it used to be and we need to develop more practical methods to handle the rounding issue. Another reason for the use of integer variables, usually binary, appears in practice because portfolio managers and their clients often have unwilling to perform small transactions. Some studies focused on models that could solve such portfolio optimization problems. Corazza and Favaretto [3] analyzed the finite divisibility of the shares using some rounding procedures. Li and Tsai [4] provided an approach based on the following three steps: (i) dual Lagrangian relaxation, (ii) linear terms transformation and (iii) separable terms transformation approaches. Bonami and Lejeune [5] presented a branch-and-bound algorithm for classical Markowitz portfolio optimization with integer constraints in which the uncertainty in estimating the expected returns is considered as part of the model. Castro et al. [6] provided a method based on the computation of some test sets using Gröbner' idea [7] for a portfolio selection model with integer variables and non-linear constraints. In the following, this paper also focuses on the new portfolio optimization with discreet variables to eliminate the gap of share rounding. We present a knapsack problem-based model to asset allocation for cases that the portfolio must include assets with relatively large values.

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On the other hand, in the real world, information on financial markets is flawed. The decision-making process should be conducted under uncertain conditions. Portfolio performance was also evaluated in such conditions in the financial market [8, 9]. Huang et al. [10] presented a combinatorial algorithm based on the possibilistic regression model to examine portfolio optimization under uncertain conditions.Gregory et al. [11] studied the uncertainty of the returns using the method proposed by Bertsimas and Sim [12, 13]. Sadjadi et al. [14] compared Ben-Tal and Nemirovski [15, 16] with Bertsimas and Sim approaches [12, 13] for analyzing the uncertainty of the returns in the portfolio optimization problem. Ghahtarani and Najafi [17] presented a goal programming approach for multi-objective portfolio optimization and examined it under uncertain conditions. Kim et al. [18] evaluated the performance of the robust optimization method based on the worst case scenarios in the stock exchanges. Maillet et al. [19] presented a new approach based on the global minimum variance to reduce the impact of uncertainty in portfolio optimization. Huang and Di [20] considered uncertain conditions for a mean-risk portfolio optimization with experts' evaluations of returns. Fernandes et al. [21] presented a one-period robust portfolio optimization with a sensory loss constraint under a data-driven polyhedral uncertainty set for adaptive asset allocation. Chen and Zhou [22] examined the mean-variance Markowitz model under uncertain condition originally developed by Ben-Tal and Nemirovski [15, 16]. Moreover, the portfolio optimization problem was examined under scenario uncertainty [23] and future returns scenarios [24]. In the following, this paper also focuses on the portfolio optimization with discreet variables under uncertain condition of Bertsimas and Sim [12, 13].

In addition, one of the primary concerns with most portfolio optimization is associated with the type of constraints considered with different models. In many cases, the resulted problem formulations do not yield practical solutions. For example, in 2006, Huang [25] examined portfolio optimization problem based on the chance constraint under fuzzy environment of returns. However, in 2010, Li et al. [26] provided more complete and accurate results by adding risk constraints and other possible conditions to the model proposed by Huang [25]. Therefore, it is necessary to apply some managerial decisions in order to make the results more practical. Hence, this paper also focuses on portfolio optimization with discrete variables and different constraints such as the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints for asset allocation under uncertain condition.

Finally, in this essay, a robust portfolio optimization based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints is presented. Then, a genetic algorithm (GA) is designed to examine the validity of the proposed model in large dimensions. A case study of the US stock market is considered to examine the performance of the proposed model and the efficacy of the designed algorithm to solve the portfolio selection model based on the knapsack problem.

The essay is organized as follows: In Section 2, robust optimization techniques are defined. In Section 3, the constrained portfolio optimization based on an improved knapsack problem is introduced under uncertain conditions. In Section 4, a meta-heuristic algorithm is designed for the proposed optimization problem. In Section 5, a real life case study of the US stock exchange and sensitivity analysis are scanned. Eventually, a synopsis of the essay and inferences are presented in Section 6.

2. PRELIMINARIES

2. 1. Robust Optimization Techniques

In the robust optimization approach, there is no assumption about the distribution of parameters and the same importance is given to all possible values for the uncertain parameters. In fact, when the robust optimization is used, the analyst follows a response that behaves well for all possible values with uncertain parameters. Soyster [27] introduced a completely conservative approach based on the worst possible condition for robust optimization. In this way, the answer to the optimization problem should remain valid for all possible values for the model parameters. Since the problem of the Soyster robust solution is that the objective function value is much worse than that of the nominal solution, Ben-Tal and Nemirovski [15, 16, 28] introduced a new approach based on ellipsoidal uncertainty collections. The conservatism of this approach is less than that of the Soyster's method and any justifiable answer from this approach is also justified in the Soyster's method. Since the Ben-Tal and Nemirovski's method is a nonlinear programming and particularly is not attractive for solving robust discrete optimization problems, Bertsimas and Sim [12, 13, 29] introduced a new robust discrete optimization problem based on the multi-level uncertainty collections. This approach takes a very low probability of occurrence of the worst condition for all parameters. To present robust optimization techniques, the following nominal linear optimization problem is considered:

$$(M_0)\begin{cases} max \ \ \sum_{j=1}^{n} c_j\, x_j \\ s.t{:}\ \sum_{j=1}^{n} a_j x_j \leq b\ , \\ x_j \geq 0 \quad \forall j \in \{1,2,\dots,n\}, \end{cases} \tag{1}$$

where a_j $(j \in \{1,2,\dots,n\})$ is assumed uncertain parameters. a_j $(\forall j \in \{1,2,\dots,n\})$ is nominal values and $\hat{a}_j$ measures the precision of the estimate. In addition, $\tilde{a}_j$ $(\forall j \in \{1,2,\dots,n\})$ is uncertain parameter defined as follows:

$$\tilde{a}_j \epsilon [a_j - \hat{a}_j, a_j + \hat{a}_j]. \tag{2}$$

Equation (2) can be written in another form as follows:

$$\tilde{a}_j \cong U(a_j - \psi_j \sigma_j, a_j + \psi_j \sigma_j) \tag{3}$$

σ_j $(\forall j \in \{1,2,\dots,n\})$ is the standard deviation of element weight j.ψ_j $(\forall j \in \{1,2,\dots,n\})$ is the coefficient of element weight j that varies around its nominal value and is defined by user. Moreover, when x^* is the optimal solution, at optimality clearly, $\theta_j = |x_j^*|$.

According to Bertsimas and Sim's method [12, 13], parameter Γ_i is defined for constraint i that limits the value of the parameter's deviation from its mean value. Γ_i is defined as follows:

$$\sum_{j=1}^{n} \eta_j \leq \Gamma_i \quad \forall i. \tag{4}$$

In fact, Γ_i is the adjustment of the stability of the proposed method versus the conservatism of the answer and takes in $[0, n]$. If $\Gamma_i = 0$, constraint$i$is analyzed in a complete deterministic state. If $\Gamma_i = n$, the parameters of constraint$i$can have the maximum fluctuation of their mean values. In other word, constraint$i$will be in the most conservative state. If $\Gamma_i \in (0, n)$, the decision-maker makes a trade- off between the protection level of the constraint $i$and the degree of conservatism of the solution. Moreover, η_j is a new stochastic variable defined as follows:

$$\eta = \{\eta \,|\, |\eta_j| \leq 1, \eta_j = (\tilde{a}_j - a_j)/\hat{a}_j, \forall j\}. \tag{5}$$

Note that $\tilde{a}_j = a_j + \eta_j \hat{a}_j$, $\sum_{j=1}^{n} \tilde{a}_j x_j$ is rewritten as follows:

$$\sum_{j=1}^{n} \tilde{a}_j x_j = \sum_{j=1}^{n} (a_j + \eta_j \hat{a}_j) x_j = \sum_{j=1}^{n} a_j x_j + \sum_{j=1}^{n} \eta_j \hat{a}_j x_j. \tag{6}$$

By using the definitions of Equations (4)-(6), M_0 (Equation (1)) can be formulated as follows:

$$\begin{cases} max \sum_{j=1}^{n} c_j x_j \\ s.t{:} \\ \sum_{j=1}^{n} a_j x_j + min \sum_{j=1}^{n} \eta_j \hat{a}_j x_j \leq b, \\ \sum_j |\eta_j| \leq \Gamma, \\ -1 \leq \eta_j \leq 1 \quad \forall j \in \{1,2,\dots n\}, \\ x_j \geq 0 \quad \forall j \in \{1,2,\dots n\}. \end{cases} \tag{7}$$

here $min \sum_{j=1}^{n} \eta_j \hat{a}_j x_j$ is equivalent to:

$$\begin{cases} -\,max \sum_{j=1}^{n} \eta_j \hat{a}_j |x_j| \\ s.t{:} \\ \sum_{j=1}^{n} \eta_j \leq \Gamma \\ 0 \leq \eta_j \leq 1 \quad \forall j \in \{1,2,\dots n\}, \\ x_j \geq 0 \quad \forall j \in \{1,2,\dots n\}. \end{cases} \tag{8}$$

Duality of Equation (8) can be formulated as follows:

$$\begin{cases} min \quad \Gamma p + \sum_{j=1}^{n} q_j \\ s.t{:} \\ p + q_j \geq e a_j |x_j{}^*| \quad \forall j \in \{1,2,\dots,n\}, \\ q_j \geq 0 \quad \forall j \in \{1,2,\dots,n\}, \\ p \geq 0, \end{cases} \tag{9}$$

Where p and q_j $(\forall j \in \{1,2,\dots,n\})$ are the dual variables. Finally, a robust optimization of M_0 (Equation (1)) using Bertsimas and Sim's method can be defined as follows:

$$(M_3)\begin{cases} max \sum_{j=1}^{n} c_j x_j \\ s.t{:} \\ \sum_{j=1}^{n} a_j x_j + \Gamma_1 p + \sum_{j=1}^{n} q_j \leq b, \\ p + q_j \geq e \hat{a}_j \theta_j \quad \forall j \in \{1,2,\dots,n\}, \\ -\theta_j \leq x_j \leq \theta_j \quad \forall j \in \{1,2,\dots,n\}, \\ x_j \geq 0 \quad \forall j \in \{1,2,\dots,n\}, \\ q_j, \theta_j \geq 0 \quad \forall j \in \{1,2,\dots,n\}, \\ p \geq 0. \end{cases} \tag{10}$$

3. PROPOSED OPTIMIZATION MODEL

3. 1. Portfolio Optimization Problem

The portfolio optimization based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints can be defined as follows:

(M_P):

$$Maximize E(R_P) = \sum_{i=1}^{n} r_i x_i \tag{11}$$

$s.t.$

$$\sum_{i=1}^{n} p_i x_i \leq B, \tag{12}$$

$$\sum_{i=1}^{n} y_i = k, \tag{13}$$

$$l_i y_i \leq x_i \leq u_i y_i, \quad \forall i \in \{1,2,\dots,n\}, \tag{14}$$

$$y_i \geq z_i, \quad \forall i \in \{1,2,\dots,n\}, \tag{15}$$

$$L_m \leq \sum_{y_i \in C_m} x_i \leq U_m, \tag{16}$$

$$x_i \in int, \quad \forall i \in \{1,2,\dots,n\}, \tag{17}$$

$$y_i \in \{0,1\}, \quad \forall i \in \{1,2,\dots,n\}. \tag{18}$$

The proposed portfolio optimization based on the improved knapsack problem tries to select a combination of all possible combinations in the portfolio that has the highest returns according to the constraints. In the other words, the objective function given in Equation (11) attempts to select a portfolio that has the maximum expected returns with respect to the constraints. $r_i, \forall i \in \{1,2,\dots,n\}$ is the return of stock and $n$is the total number of stocks. $x_i, \forall i \in \{1,2,\dots,n\}$ is the integer variabel that demonstrates the number of n per shares. One of the realistic aspects of the portfolio optimization problem is considering the budget ceiling. Equation (12) introduces the budget constraint that states the sum of the weights of the total assets is less than or equal to the financial plan. p_i is the price of stock i, $B$is the total available budget and x_i is the integer variable that represents the number of stock i.

Equation (13) introduces the cardinality constraint that specifies the number of shares invested in the portfolio to provide the model according to the investor's expectations.k is the number of shares allowed to appear in the portfolio and$y_i$is the binary variable signaling whether asset$i$is involved in the portfolio or not.$y_i = 1$, if asset i is involved in the portfolio, and$y_i = 0$in any other way.

Equation (14) explains the floor and ceiling constraints that specifies the maximum and minimum ratio of investment per share. l_i and u_i are the lower and upper bound of stock i, respectively. Therefore, Equation (14) states that the number of per share should only fluctuate between its lower and upper bound [30].

Equation (15) introduces the pre-assignment constraint that describes the investor's subjective preferences. A stockholder may intuitively prefer a particular set of securities to be involved in the portfolio, with its proportion either fixed or to be specified. z_i is the binary vector indicating whether stock i is included in the pre-assigned set $\{Z\}$ or not that has to be involved in the portfolio or not.

Equation (16) introduces the class constraints [31], when all available shares can be divided into several categories based on various features or according to the investor's own intuition or preference (such as energy assets and transportation assets), these constraints are used to limit the proportion of the portfolio that must be invested in a particular category of stocks. In practical terms, stockholder may also want to confine the whole proportion invested in per class to a defined class limit [32]. U_m and L_m are the lower and upper proportion limit for class m, respectively.

Equations (17) and (18) explain the structure of the decision variables that are binary and integer variables. The main difference between the proposed model based on the improved knapsack problem with the classical mean-variance model and its developed models is the type of decision variable. The classical mean-variance model is based on the continuous variables. One of the primary concerns with Markowitz asset allocation is that the optimal result may not be necessarily applicable for asset allocation when the cost of purchasing specific shares which their value is relatively high, e.g. Berkshire Hathaway Inc. Representative sample of four shares in a portfolio with due regard the budget allocated to these four shares is one million dollars and market prices are 340,000, 1500, 250 and \$1600, respectively. The optimal weights based on the Markowitz model are of 0.2, 0.4, 0.1, and 0.3. Eventually, the number of shares is 0.588, 266.6, 400 and 187.5, respectively. It goes without saying, the numerical quantity of second and forth shares are not a whole number. Consequently, the rounding problem is getting more important than used to be and we need to develop more practical methods to handle the rounding issue. In addition, the numerical quantity of first share (0.588) is less than one share. This result illustrates that to solve portfolio selection models including specific shares which their value is relatively high, it is necessary to use portfolio optimization with integer variables. Hence, the portfolio optimization based on an improved knapsack problem is presented by considering discrete (integer and binary) variables. Since the proposed model is the mixed- integer programming, it has the potency to assign the optimal number to each selected share as an integer number and provide an acceptable solution for specific shares whose value is relatively high.

In the following, the uncertain condition is considered in the proposed portfolio selection model.

3. 2. Robust Portfolio Optimization In the real world, information on financial markets is flawed and the decision-making process should be conducted under uncertain condition. The unreliability of the parameters is explained by uncertain set, which involves all possible values for parameters. Since the proposed model is the mixed - integer programming, the robust approach of Bertsimas and Sim [12, 13] is used. Therefore, a robust portfolio optimization based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints can be defined as follows:

(M_R):

$$Maximiz\ R_P \tag{19}$$

$s.t$:

$$R_P - \sum_{j=1}^{n} r_j x_j + \Gamma_1 \lambda_1 + \sum_{j=1}^{n} \delta_{1j} \leq 0, \quad (20)$$

$$\sum_{j=1}^{n} p_j x_j + \Gamma_2 \lambda_2 + \sum_{j=1}^{n} \delta_{2j} \leq B, \quad (21)$$

$$\lambda_1 + \delta_{1j} \geq e\hat{r}_j \theta_j \quad \forall j \in \{1,2,\ldots,n\}, \quad (22)$$

$$\lambda_2 + \delta_{2j} \geq e\hat{p}_j \theta_j \quad \forall j \in \{1,2,\ldots,n\}, \quad (23)$$

$$-\theta_j \leq x_j \leq \theta_j \quad \forall j \in \{1,2,\ldots,n\}, \quad (24)$$

$$\sum_{j=1}^{n} y_j = k, \quad (25)$$

$$l_j y_j \leq x_j \leq u_j y_j \quad \forall j \in \{1,2,\ldots,n\}, \quad (26)$$

$$y_j \geq z_j \quad \forall j \in \{1,2,\ldots,n\}, \quad (27)$$

$$L_m \leq \sum_{y_j \in C_m} x_j \leq U_m \quad (28)$$

$$x_j \in int \quad \forall j \in \{1,2,\ldots,n\}, \quad (29)$$

$$y_j \in \{0,1\} \quad \forall j \in \{1,2,\ldots,n\}, \quad (30)$$

$$\delta_{1j}, \delta_{2j}, \theta_j \geq 0 \quad \forall j \in \{1,2,\ldots,n\}, \quad (31)$$

$$\lambda_1, \lambda_2 \geq 0. \quad (32)$$

Equations (19) and (20) define the objective function that maximizes the expected return of the portfolio according to the robust optimization method of Bertsimas and Sim. In the proposed constrained portfolio selection model is based on an improved knapsack problem, the parameters related to the price of each stock $(p_j, \forall j \in \{1,2,\ldots,n\}) \oplus$ and the return of each stock $(r_j, \forall j \in \{1,2,\ldots,n\})$ are considered as uncertain parameters. Thus, the uncertain parameter of stock price j $(\tilde{p}_j)$ takes values in $[p_j - \hat{p}_j, p_j + \hat{p}_j]$, where p_j is the nominal value of stock price j and $\hat{p}_j$ measures the precision of the estimate. The uncertain parameter of stock return j $(\tilde{r}_j)$ takes values in $[r_j - \hat{r}_j, r_j + \hat{r}_j]$, where r_j is the nominal value of stock return j and $\hat{r}_j$ measures the precision of the estimate. Therefore, Γ_1 and Γ_2 are defined as the budget of uncertainty in Equations (20) and (21), respectively. Γ_i is the adjustment of the stability of the proposed method versus the conservatism of the answer. Moreover, Γ_1 and Γ_2 are not necessarily an integer number and take values in $[0, n]$. In addition, according to the robust technique of Bertsimas and sim, λ_1, δ_{1j} and $\lambda_2, \delta_{2j} \forall j \in \{1,2,\ldots,n\}$ are the dual variables in Equations (22) and (23), respectively. When x^* is the optimal solution, at optimality clearly, $\theta_j = |x_j^*|$ (Equation (24)). Equation (25) is the cardinality constraint, Equation (26) is the floor and ceiling constraints, Equation (27) is the pre-assignment constraint and Equation (28) is the class constraints and class limit constraints. These constraints are fully explained in Section (3.1). Moreover, Equations (29)-(32) explain the structure of the decision variables. Eventually, in the following, the aspects and benefits of the proposed model are listed.

- The rounding problem is getting more important than it used to be and we need to develop more practical methods to handle the rounding issue. Hence, this paper also focuses on portfolio optimization with discrete variables to eliminate the gap of share rounding.
- The proposed portfolio selection model is based on discrete variables that allow the treatment of some portfolio optimization problems in a more realistic way and provide the possibility of adding some natural aspects to the model.
- Since the proposed model is the mixed- integer programming, it has the potency to assign the optimal number to each selected share as an integer number.
- The proposed portfolio optimization provides an acceptable solution for specific shares whose value are relatively high.
- The proposed portfolio selection model considers many limitations on real financial markets.
- One of the primary concerns with most portfolio optimization is associated with the type of constraints considered with different models. In many cases, the resulted problem formulations do not yield in practical solutions. Therefore, it is necessary to apply some managerial decisions in order to make the results more practical. This paper presents a portfolio optimization based on an improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints for asset allocation.
- In the portfolio optimization based on the improved knapsack problem, the uncertain condition of the parameters is regarded in the target function and limitation at the same time.
- Since the proposed portfolio optimization is the mixed-integer programming, the robust approach of Bertsimas and Sim [12, 13] is used.
- Finally, the robust knapsack based constrained portfolio optimization selects the best portfolio based on maximum returns.

4. GENETIC ALGORITHM

A knapsack problem is a combinatorial optimization problem [33,34] and one of the classical NP-complete problems [35]. Since the proposed portfolio optimization is based on the knapsack problem, it is essential to solve a wide-ranging problem, applying an approximate algorithm that presents a near-optimal answer. GA is believed to be one of the most popular techniques for solving large scale mixed-integer programming. Khuri et al. [36] illustrated that GA can provide a proper solution for NP-complete problems such as the 0/1 multiple knapsack problem (MKP). In GA, first several random or algorithmic responses are generated for the problem. Each answer is called a chromosome and the answer set is called primitive population. Then, each chromosome will be assigned to the value of the objective function. Fitness function is used for choosing more suitable chromosomes. These chosen chromosomes are combined and mutated with use of the operators. Ultimately, the current population is combined with a new population that results from the combination and mutation in the chromosomes. This process is repeated until the final condition is provided [37]. The main steps of a simple GA can be found in the literature [31]. Algorithm 1 shows the stages of genetic algorithm:

Algorithm 1. Pseudocode of GA

Inputs
ASSIGN value: population size, crossover rate, mutationrate, number of iterations;
INITIAL population*:* the initial random population of individuals;
EVALUATE fitness function: fitness function assesses whether fits or not;
Repeat
SELECT individuals from the next generation population:
(Using roulette wheel selection and regarding all constraints);
RECOMBINE pairs of parents (single point crossover with selected probability);
MUTATE the offspring (swap mutation for permutation with selected probability);
EVALUATE whether new individuals fit or not;
REPLACE some or all of the population by the children;
UNTIL a satisfactory solution has been found based on the fitness function;
END.

In the following, each section is separately described:

- Generic encoding: The representation is the first step to solve the problems with meta-heuristic algorithms. In genetic algorithms, each chromosome illustrates a point in the search space and a possible solution to the problem. The chromosome of this study includes a matrix$(2 \times n)$. The first row contains a binary (zero or one) string that shows the selected stocks. *"1"* is assigned to stocks that are selected and *"0"* is assigned to stocks that are not selected. The second row contains a string of integers that represent the number of stocks that are between their upper and lower bounds. Figure 1 shows a simple example of the representation for encoding the portfolio optimization problem:
- Fitness function: A fitness function has to be used to measure the fitness value of each individual chromosome within the population of every generation. For each chromosome, a fitness function returns a number that indicates the suitability or individual ability of that chromosome. In this study, the fitness function is defined as the total returns, which must have the possible maximum value (Equation (11)).
- Handling of configuration limitations: Moreover, to achieve possible results, each chromosome has to convince the existing limitations. A number of ways to handle constraints have been introduced in the literature, such as repairing, special representations, variable restriction, penalty function and modifying generic operator methods [38,39]. In this study, the repair mechanism, special representations and variable restriction are applied for constraints. For instance, Equations (12) and (14) are repaired between their lower and upper bounds, if they exceed their limit. Equations (13) and (15) have special binary representations and generate valid solutions. Equations (17) and (18) have variable restriction, which randomly generate the integer and binary numbers, respectively. Moreover, if Equation (16) is not provided, the answers are generated from the beginning.
- Initial population: Initialization includes producing primary solutions to the optimization problem. The primary solutions can be created either randomly or using some heuristic approaches [40]. In this research, according to the generic encoding, the initial population is selected randomly. Then, this initial random population is checked in all constraints based on handling of configuration constraints. Finally, the algorithm begins with a valid response.
- Selection and reproduction: In this research, the roulette wheel selection method is used. The roulette wheel method is one of the most suitable random selection operators that are filled with the cumulative probabilities based on the fitness function of each chromosome. The more elegant chromosomes, the more likely they will be selected for reproduction.

	S_1	**S_2**	**S_3**	**…**	**S_{n-2}**	**S_{n-1}**	**S_n**
$y_i = \{y_1, y_2, \ldots, y_n\} \in \{0,1\}$	1	0	1	…	0	1	0
$x_i = \{x_1, x_2, \ldots, x_n\} \in int$	5	3	10	…	6	12	9

Figure 1. Representation of chromosome in GA

Crossover and mutation: The purpose of this process is to maintain the crowd diversity and variety and also to prevent early and incomplete convergence at a local optimum. But this process should not always occur because it creates a purely random search in the search space. In this research, a single point crossover method is used. In this operator, a random position is considered between two genes in the parent chromosomes. A new pair of chromosomes is obtained by changing all genes in the right or left side of this position. After the crossover operation is completed, the mutation operator acts on the chromosomes. In this research, swap mutation for permutation is used [41]. This operator randomly selects two genes from a chromosome and then changes them together. Figures 2 and 3 show a simple example of the single point crossover and the swap mutation, respectively.

The purpose of the crossover and mutation are to maintain the crowd diversity and variety and also to prevent early and incomplete convergence at a local optimum. But this process should not always occur because it creates a purely random search in the search space [42].

5. CASE STUDY: APPLICATION TO THE STOCKS OF US

5. 1. Numerical Analysis The numerical analysis of the proposed study is associated with the shares of the NASDAQ stock exchanges, the Dow Jones Industrial Average (DJIA) and S&P 500 index listed in US stock market and includes daily financial time series data from February 2, 2014 to February 2, 2019; totaling 1259 trading days. The information about the price and returns of any stocks and their upper and lower bounds that are used in this research are available in Appendix A. Results are obtained for the constrained portfolio selection model with cardinality $k = \{5,6,7,8,9,10\}$, uncertain paremeters $\Gamma_i = \{5,6,7,8,9,10\}$, pre-assignment $Z = \{3\}$, class $M = 3$ with 10, 10, 10 assets in each class (i.e., $C_1 \in \{s_1, \dots, s_{10}\}$, $C_2 \in \{s_{11}, \dots, s_{20}\}$, $C_3 \in \{s_{21}, \dots, s_{30}\}$), and $L_m = 8$, $U_m = 200$ for each $m = 1,2,3$. In addition, the degree of confidence is considered 95%.

Parent chromosomes:

A	0	1	0	1	0	1	1
	4	5	1	9	6	2	7
B	1	1	1	0	0	1	0
	1	8	4	1	6	1	3

offspring chromosomes:

0	1	1	0	0	1	0
4	5	4	1	6	1	3
1	1	0	1	0	1	1
1	8	1	9	6	2	7

Figure 2. Single point crossover

Parent chromosome:

0	1	0	1	0	1	1
4	5	1	9	6	2	7

offspring chromosome:

0	0	0	1	1	1	1
4	6	1	9	5	2	7

Figure 3. Swap mutation

First, the proposed model using deterministic parameters is solved with GAMS software. Given that the portfolio optimization based on an improved knapsack problem is the mixed-integer programming and the solutions are based on the assumed budget. Therefore, the GAMS software has the capability to specify the optimal portfolio with cardinality $k = \{5,6,7,8,9,10\}$ and cannot solve this problem in larger dimensions.

Then, the robust optimization model (M_R) based on the different values of uncertain parameters $\Gamma_i = \{5,6,7,8,9,10\}$ and different cardinalities $k = \{5,6,7,8,9,10\}$ is solved with GAMS software. It should be noted that the uncertain parameters take values in $\Gamma_i \in (0k]$ for each k. In fact, $\Gamma_i = 0$ represents results in nominal solution and $\Gamma_i = k$ represents the worst case scenario where uncertainty exists among all input parameters. Table 1 shows the results of solving the certain and robust optimization model for different cardinalities $k = \{5,6,7,8,9,10\}$.

As shown in Table 1, the objective function, which is the maximum returns, has a declining trend by increasing the uncertainty parameter in proposed optimization problem with different cardinalities, which means that the value of the target function decreases with an increase in risk. This process reflects the validity of the proposed model.

Table 1 illustrates the important role of risk assessment in the proposed model and different results are obtained by considering various values for the uncertain parameters. The mean of these solutions is significantly different from the nominal values. For instance, the expected return of the proposed model with cardinality $k = 10$ and uncertain parameters $\Gamma_i \in [010]$ lies in [3.22240 3.92010] where $\Gamma_i = 0$ demonstrates the expected return of the portfolio in nominal solution that is 3.92010 and $\Gamma_i = 10$ demonstrates the expected return of the portfolio based on the worst case scenario among all input parameters that is 3.22240. The standard error of mean and standard deviation of these two solutions are 0.349 and 0.493, respectively. Other values of Γ_i result in the expected return between 3.22240 to 3.92010. Moreover, the selected shares of the portfolio are different based on the different uncertain parameters (except stocks that include pre-assignment constraint).

In the following, the consequences of the exact solution should be contrasted with the consequences of the proposed GA in small dimensions to measure the validity of the

TABLE 1. Exact solutions of the certain and robust model

	$\boldsymbol{k=5}$	$\boldsymbol{k=6}$	$\boldsymbol{k=7}$	$\boldsymbol{k=8}$	$\boldsymbol{k=9}$	$\boldsymbol{k=10}$
$\boldsymbol{\Gamma_i=0}$	E(R)= 5.35853 $X_7=8$, $X_{12}=12$ $X_{19}=19$, $X_{24}=40$ $X_{29}=8$	E(R)= 5.26656 $X_5=10$, $X_7=8$ $X_{12}=15$, $X_{19}=10$ $X_{24}=40$, $X_{29}=8$	E(R)= 5.13979 $X_5=11$, $X_7=8$ $X_{10}=11$, $X_{19}=9$ $X_{24}=40$, $X_{28}=7$ $X_{29}=8$	E(R)= 4.78896 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=36$ $X_{28}=7$, $X_{29}=8$	E(R)= 4.35559 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=28$ $X_{26}=1$, $X_{28}=7$ $X_{29}=8$	E(R)= 3.92010 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{10}=11$, $X_{19}=8$ $X_{24}=20$, $X_{26}=1$ $X_{28}=7$, $X_{29}=8$
$\boldsymbol{\Gamma_i=5}$	E(R)=4.83100 $X_7=8$, $X_{12}=12$ $X_{19}=16$, $X_{24}=40$ $X_{29}=8$	E(R)=4.80158 $X_5=10$, $X_7=8$ $X_{12}=12$, $X_{19}=10$ $X_{24}=39$, $X_{29}=8$	E(R)=4.66865 $X_5=10$, $X_7=8$ $X_{10}=11$, $X_{19}=8$ $X_{24}=38$, $X_{28}=7$ $X_{29}=8$	E(R)=4.31243 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=31$ $X_{28}=7$, $X_{29}=8$	E(R)=3.90130 $X_5=10$, $X_7=8$ $X_9=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=23$ $X_{26}=1$, $X_{28}=7$ $X_{29}=8$	E(R)=3.35206 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{10}=11$, $X_{11}=1$ $X_{19}=8$, $X_{24}=21$ $X_{26}=1$, $X_{29}=8$
$\boldsymbol{\Gamma_i=6}$	-	E(R)= 4.76082 $X_5=10$, $X_7=8$ $X_{12}=14$, $X_{19}=8$ $X_{24}=40$, $X_{29}=8$	E(R)= 4.59583 $X_5=10$, $X_7=8$ $X_{10}=11$, $X_{19}=8$ $X_{24}=37$, $X_{28}=7$ $X_{29}=8$	E(R)= 4.23961 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=30$ $X_{28}=7$, $X_{29}=8$	E(R)= 3.83289 $X_5=10$, $X_7=8$ $X_8=1$, $X_{19}=8$ $X_{24}=22$, $X_{26}=1$ $X_{27}=9$, $X_{28}=7$ $X_{29}=8$	E(R)= 3.33038 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{11}=1$, $X_{19}=8$ $X_{24}=22$, $X_{26}=1$ $X_{28}=7$, $X_{29}=8$
$\boldsymbol{\Gamma_i=7}$	-	-	E(R)= 4.58944 $X_5=10$, $X_7=8$ $X_{10}=11$, $X_{19}=8$ $X_{24}=37$, $X_{28}=7$ $X_{29}=8$	E(R)= 4.21518 $X_5=10$, $X_7=8$ $X_{10}=11$, $X_{19}=8$ $X_{24}=20$, $X_{27}=9$ $X_{28}=7$, $X_{29}=8$	E(R)= 3.82209 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=22$ $X_{26}=1$, $X_{28}=7$ $X_{29}=8$	E(R)= 3.29217 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{10}=11$, $X_{11}=1$ $X_{19}=8$, $X_{24}=20$ $X_{26}=1$, $X_{29}=8$
$\boldsymbol{\Gamma_i=8}$	-	-	-	E(R)= 4.17971 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=29$ $X_{28}=7$, $X_{29}=8$	E(R)= 3.76858 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=21$ $X_{26}=1$, $X_{28}=7$ $X_{29}=8$	E(R)= 3.27627 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{11}=1$, $X_{19}=8$ $X_{24}=21$, $X_{26}=1$ $X_{28}=7$, $X_{29}=8$
$\boldsymbol{\Gamma_i=9}$	-	-	-	-	E(R)= 3.76801 $X_5=10$, $X_7=8$ $X_8=1$, $X_{10}=11$ $X_{19}=8$, $X_{24}=21$ $X_{26}=1$, $X_{28}=7$ $X_{29}=8$	E(R)= 3.22287 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{11}=1$, $X_{19}=8$ $X_{24}=20$, $X_{26}=1$ $X_{28}=7$, $X_{29}=8$
$\boldsymbol{\Gamma_i=10}$	-	-	-	-	-	E(R)= 3.22240 $X_2=1$, $X_5=10$ $X_7=8$, $X_8=1$ $X_{11}=1$, $X_{19}=8$ $X_{24}=20$, $X_{26}=1$ $X_{28}=7$, $X_{29}=8$

proposed GA for its use in larger dimensions. Hence, the Taguchi method is applied to choose proper parameters.

Taguchi approach is applied for four parameters of the proposed GA (number of iterations, population size, mutation rate, crossover rate) at three different levels. $\{200,500,1000\}$ is employed for the number of iterations and the population size; 0.05, 0.5 and 0.2 for the mutation rate; 0.8, 1 and 0.7 for the crossover rate.

Table 2 presents the parameter setting of the implemented GA for the certain and robust model.

TABLE 2. Parameter setting of GA

Parameter	Certain optimization model	Robust optimization model
Number of iterations	500	500
Population size	1000	500
Crossover method	Single point crossover $Pc = 1.0$	Single point crossover $Pc = 1.0$
Mutation method	Swap mutation for permutation $Pm = 0.05$	Swap mutation for permutation $Pm = 0.2$

Table 3 presents the results of the five-time implementation of the proposed GA for the certain optimization model ($\Gamma_i = 0$) with different cardinalities $k = 5,6,7,8,9,10$. Then, Table 4 and Figure 4 presents a comparison of the exact solutions with GA solutions in the certain optimization model ($\Gamma_i = 0$) with different cardinalities $k = 5,6,7,8,9,10$.

Table 5 gives the results of the five-time implementation of the proposed GA for the robust optimization model with different cardinalities $k = 5,6,7,8,9,10$ and uncertain parameters $\Gamma_i = 5$. Then, Table 6 and Figure 5 presents a comparison of the exact solutions with GA solutions in the robust optimization model with different cardinalities $k = 5,6,7,8,9,10$ and uncertain parameters $\Gamma_i = 5$.

As can be seen in Table 4, the standard errors of mean (SE. Mean), standard deviations (S.D.) and probability values (P-value) are less than 0.0151, 0.0214 and 0.002, simultaneously. In addition, in TABLE 6, SE. Mean, S.D. and P-value are less than 0.0267, 0.0377, and 0.004, simultaneously.

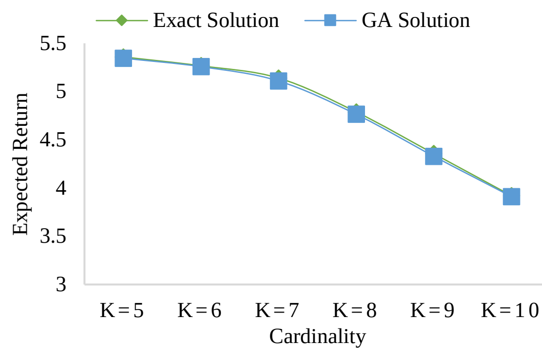

Figure 4. Comparison of the exact solutions with GA solutions for the certain optimization model

TABLE 3. GA solutions of the certain optimization model

No.	Size					
	K=5	**K=6**	**K=7**	**K=8**	**K=9**	**K=10**
1	5.3313	5.2665	5.1096	4.7889	4.3513	3.9200
2	5.3585	5.2609	5.1250	4.7425	4.3207	3.8976
3	5.3384	5.2652	5.0960	4.7379	4.3112	3.9173
4	5.3449	5.2582	5.1232	4.7639	4.3046	3.9012
5	5.3558	5.2395	5.0935	4.7868	4.3555	3.9200
Mean	5.3458	5.2581	5.1095	4.7641	4.3287	3.9113
SE. Mean	0.0051	0.0048	0.0065	0.0107	0.0105	0.0048
S.D.	0.0115	0.0109	0.0147	0.0239	0.0234	0.0109

TABLE 4. Compression of the exact results with GA results for the certain optimization model

	Size					
	K=5	**K=6**	**K=7**	**K=8**	**K=9**	**K=10**
Exact	5.3585	5.2665	5.1397	4.7889	4.3555	3.9201
GA	5.3458	5.2581	5.1095	4.7641	4.3287	3.9113
SE. Mean	0.0063	0.0042	0.0151	0.0124	0.0134	0.0044
S.D.	0.0090	0.0059	0.0214	0.0176	0.0190	0.0063
P-Value*	0.001	0.001	0.002	0.002	0.002	0.001

* denotes rejection of the hypothesis at the 0.01 level

TABLE 5. GA solutions for the robust optimization model

		Size					
		K=5	**K=6**	**K=7**	**K=8**	**K=9**	**K=10**
	1	4.7529	4.7423	4.6486	4.2628	3.8800	3.3204
	2	4.7919	4.7608	4.5762	4.2441	3.8678	3.3249
$\Gamma = 5$	3	4.8284	4.7357	4.6576	4.2548	3.8996	3.2950
	4	4.7912	4.7570	4.6422	4.2461	3.8745	3.3518
	5	4.8310	4.7450	4.6187	4.3068	3.8658	3.2921
Mean		4.7991	4.7482	4.6287	4.2629	3.8775	3.3168
SE.Mean		0.0144	0.0046	0.0146	0.0115	0.0060	0.0109
S.D.		0.0321	0.0105	0.0327	0.0256	0.0136	0.0244

TABLE 6. Compression of the exact solutions with GA solutions for the robust optimization model

	Size					
	K=5	**K=6**	**K=7**	**K=8**	**K=9**	**K=10**
Exact	4.8310	4.8015	4.6686	4.3124	3.9013	3.3520
GA	4.7991	4.7482	4.6287	4.2629	3.8775	3.3168
SE.Mean	0.0160	0.0267	0.0199	0.0248	0.0119	0.0176
S.D.	0.0226	0.0377	0.0282	0.0350	0.0168	0.0249
P-Value*	0.002	0.004	0.003	0.004	0.002	0.003

* denotes rejection of the hypothesis at the 0.01 level

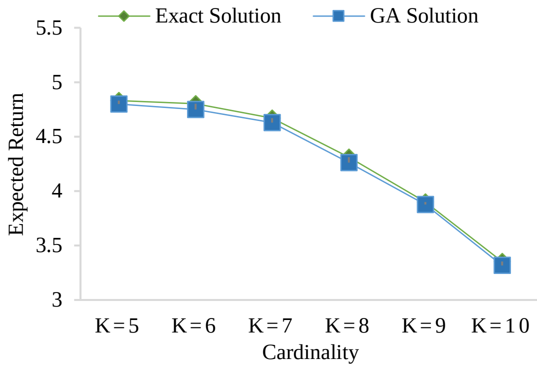

Figure 5. Comparison of the exact solutions with GA solutions for the robust optimization model

Finally, all results show that the used GA is very suitable for solving the proposed portfolio selection model in small dimensions. All results confirm the reliability and credibility of the used GA for the suggested model. Hence, the used GA can be cited in larger dimensions as well.

6. CONCLUSION

One of the main concerns with Markowitz asset allocation is that the optimal result may not be necessarily applicable for asset allocation when the cost of purchasing specific shares which their value is relatively high. In this essay, we have suggested the portfolio optimization based on the improved knapsack problem with the cardinality, floor and ceiling, budget, class, class limit and pre-assignment constraints for asset allocation. To handle uncertainty associated with different parameters of the proposed model, we have used robust optimization techniques.

The proposed model was investigated with some realistic data from US stock market. The portfolio optimization based on an improved knapsack problem was solved using GAMS software in small dimensions. Since the complexity of the knapsack problem is NP-complete, it is not possible to use common mathematical and exact methods to reach the optimal answer in large dimensions. Therefore, the proposed model was solved using GAin larger scales. The results of this research show that the robust portfolio optimization model has high reliability and efficiency in stock market optimization and the used GA is valid for solving this model in large dimensions.

Since the proposed model based on the improved knapsack problem is the mixed- integer programming, it has the potency to assign the optimal number to each selected share as an integer number and provides an acceptable solution for specific shares whose value is relatively high. Finally, due to the limitations of real financial markets, the proposed model maximizes the returns of investment and considers the uncertain conditions, simultaneously.

7. APPENDIX A

data. Data related to this paper can be found at https://my.pcloud.com/publink/show?code=XZ5WWhkZAv23S9ISmyYd8ljhFUgLfQSNnkkV

8. REFERENCE

1. Markowitz, H., “Portfolio selection”, ***The Journal of Finance***, Vol. 7, No. 1, (1952), 77–91.
2. Markowitz, H. M., “Portfolio Selection: Efficient Diversification of Investment”, John Wiley & Sons, New York, USA, (1959).
3. Corazza, M. and Favaretto, D., “On the existence of solutions to the quadratic mixed-integer mean–variance portfolio selection problem”, ***European Journal of Operational Research***, Vol. 176, No. 3, (2007), 1947–1960.
4. Li, H.L. and Tsai, J.F., “A distributed computation algorithm for solving portfolio problems with integer variables”, ***European Journal of Operational Research***, Vol. 186, No.2, (2008),882–891.
5. Bonami, P. and Lejeune, M. A., “An exact solution approach for portfolio optimization problems under stochastic and integer constraints”, ***Operations Research***, Vol. 57, No. 3, (2009), 650–670.
6. Castro, F., Gago, J., Hartillo, I., Puerto, J., and Ucha, J. M., “An algebraic approach to integer portfolio problems”, ***European Journal of Operational Research***, Vol. 210, No.3, (2011), 647–659.
7. Adams, W. W., Adams, W. W., Loustaunau, P., ADAMS, W. H., and Adams, W. W., “An Introduction to Gröbner Bases”, American Mathematical Soc, (1994).
8. Zymler, S., Rustem, B., and Kuhn, D., “Robust portfolio optimization with derivative insurance guarantees”, ***European Journal of Operational Research***, Vol. 210, No.2, (2011), 410–424.
9. Scutellà, M. G. and Recchia, R., “Robust portfolio asset allocation and risk measures”, ***Annals of Operations Research***, Vol. 204, No.1, (2013), 145–169.
10. Huang, J.-J., Tzeng, G.-H., and Ong, C.-S., “A novel algorithm for uncertain portfolio selection”, ***Applied Mathematics and computation***, Vol. 173, No.1, (2006), 350–359.
11. Gregory, C., Darby-Dowman, K., and Mitra, G., “Robust optimization and portfolio selection: The cost of robustness”, ***European Journal of Operational Research***, Vol. 212, No.2, (2011), 417–428.
12. Bertsimas, D. and Sim, M., “Robust discrete optimization and network flows”, ***Mathematical Programming***, Vol. 98, No. 1–3, (2003), 49–71.
13. Bertsimas, D. and Sim, M., “The price of robustness”, ***Operations Research***, Vol. 52, No.1, (2004), 35–53.
14. Sadjadi, S. J., Gharakhani, M., and Safari, E., “Robust optimization framework for cardinality constrained portfolio problem”, ***Applied Soft Computing***, Vol. 12, No. 1, (2012), 91–99.
15. Ben-Tal, A. and Nemirovski, A., “Robust convex optimization”, ***Mathematics of Operations Research***, Vol. 23, No. 4, (1998), 769–805.
16. Ben-Tal, A. and Nemirovski, A., “Robust solutions of uncertain

linear programs", ***Operations Research Letters***, Vol. 25, No. 1, (1999), 1–13.

17. Ghahtarani, A. and Najafi, A.A., "Robust goal programming for multi-objective portfolio selection problem", ***Economic Modelling***, Vol. 33, (2013), 588–592.
18. Kim, W. C., Kim, J. H., Mulvey, J. M., and Fabozzi, F. J., "Focusing on the worst state for robust investing", ***International Review of Financial Analysis***, Vol. 39, (2015), 19–31.
19. Maillet, B., Tokpavi, S., and Vaucher, B., "Global minimum variance portfolio optimisation under some model risk: A robust regression-based approach", ***European Journal of Operational Research***, Vol. 244, No. 1, (2015), 289–299.
20. Huang, X. and Di, H., "Uncertain portfolio selection with background risk", ***Applied Mathematics and Computation***, Vol. 276, (2016), 284–296.
21. Fernandes, B., Street, A., Valladão, D., and Fernandes, C., "An adaptive robust portfolio optimization model with loss constraints based on data-driven polyhedral uncertainty sets", ***European Journal of Operational Research***, Vol. 255, No. 3, (2016), 961–970.
22. Chen, C. and Zhou, Y., "Robust multiobjective portfolio with higher moments", ***Expert Systems with Applications***, Vol. 100, (2018),165–181.
23. Xidonas, P., Hassapis, C., Soulis, J., and Samitas, A., "Robust minimum variance portfolio optimization modelling under scenario uncertainty", ***Economic Modelling***, Vol. 64, (2017), 60–71.
24. Xidonas, P., Mavrotas, G., Hassapis, C., and Zopounidis, C., "Robust multiobjective portfolio optimization: A minimax regret approach", ***European Journal of Operational Research***, Vol. 262, No. 1, (2017), 299–305.
25. Huang, X., "Fuzzy chance-constrained portfolio selection", ***Applied Mathematics and Computation***, Vol. 177, No. 2, (2006),500–507.
26. Li, X., Qin, Z., and Yang, L., "A chance-constrained portfolio selection model with risk constraints", ***Applied Mathematics and Computation***, Vol. 217, No. 2, (2010), 949–951.
27. Soyster, A. L., "Convex programming with set-inclusive constraints and applications to inexact linear programming", ***Operations research***, Vol. 21, No. 5, (1973), 1154–1157.
28. Ben-Tal, A. and Nemirovski, A., "Robust solutions of linear programming problems contaminated with uncertain data", ***Mathematical Programming***, Vol. 88, No. 3, (2000), 411–424.
29. Bertsimas, D. and Sim, M., "Tractable approximations to robust conic optimization problems", ***Mathematical Programming***, Vol. 107, No. 1–2, (2006), 5–36.
30. Vaezi, F., Sadjadi, S. J., and Makui, A., "A portfolio selection model based on the knapsack problem under uncertainty", ***PLOS One***, Vol. 14, No. 5, (2019), 1-19.
31. Chang, T.J., Meade, N., Beasley, J. E., and Sharaiha, Y. M., "Heuristics for cardinality constrained portfolio optimisation", ***Computers & Operations Research***, Vol. 27, No. 13, (2000), 1271–1302.
32. Anagnostopoulos, K. P. and Mamanis, G., "Multiobjective evolutionary algorithms for complex portfolio optimization problems", ***Computational Management Science***, Vol. 8, No. 3, (2011), 259–279.
33. Lenstra, J. K. and Kan, A. R., "Computational complexity of discrete optimization problems", ***Annals of Discrete Mathematics***,Vol. 4, (1979), 121–140.
34. Papadimitriou, C. H., "On the complexity of integer programming", ***Journal of the Assoclauon for Compuung Machinery***, Vol. 28, No. 4, (1981) 765–768.
35. Kellerer, H., Pferschy, U., and Pisinger, D., "Introduction to NP-Completeness of knapsack problems", In: Knapsack Problems, Springer, Berlin, Heidelberg, (2004), 483–493.
36. Khuri, S., Bäck, T., and Heitkötter, J., "The zero/one multiple knapsack problem and genetic algorithms", In Proceedings of the 1994 ACM Symposium on Applied Computing, ACM, (1994), 188–193.
37. Holland, J.H., "Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence", MIT press, (1992).
38. Gen, M. and Cheng, R., "Genetic Algorithms and Engineering Optimization", John Wiley & Sons, (2000).
39. Coello, C.A.C., "Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art", ***Computer Methods in Applied Mechanics and Engineering***, Vol. 191, No. 11–12, (2002), 1245–1287.
40. Mitchell, M., "An Introduction to Genetic Algorithms", MIT press, (1998).
41. Sivanandam, S.N. and Deepa, S.N., "Genetic algorithms", In: Introduction to Genetic Algorithms, Springer, Berlin, Heidelberg, (2008), 15–37.
42. Back, T., "Evolutionary Algorithms in Theory and Practice: Evolution Strategies, Evolutionary Programming, Genetic Algorithms", Oxford university press, (1996).

Persian Abstract

چکیده

یکی از نگرانی‌های اساسی در بهینه‌سازی سبد سرمایه‌گذاری، تعیین تعداد سهام برای دارایی‌هایی با ارزش خالص نسبتاً بالا در بازار سهام است .روش‌های سنتی تخصیص دارایی‌ها، مانند قضیه مارکویتز راه‌حل را به عنوان درصد ارائه می‌دهند و این نسبت ممکن است نشان‌دهنده تقسیم نیمی از سهم در بازار باشد که در واقعیت، عملی نمی‌باشد. بنابراین، لازم است روشی برای تعیین تعداد بهینه سهام برای هر دارایی پیشنهاد شود .همچنین، یکی دیگر از اصلی‌ترین نگرانی‌ها، مربوط به در نظر گرفتن محدودیت‌های مختلف بطور همزمان در مدل‌ها است .در این مقاله یک مدل بهینه‌سازی سبد سرمایه‌گذاری بر اساس مسئله کوله‌پشتی با در نظر گرفتن محدودیت‌های کاردینالیتی، کف و سقف، بودجه، کلاس، کلاس محدود شده و محدودیت‌های پیش تعیین‌شده برای تخصیص دارایی‌ها تحت شرایط عدم قطعیت ارائه می‌شود .سرانجام، یک الگوریتم ژنتیک برای یافتن راه‌حل‌ها در ابعاد بزرگ طراحی شده است. مطالعه موردی بر روی تعدادی از سهام موجود در بورس اوراق بهادار ایالات متحده انجام شده است.