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Integrated Inspection Planning and Preventive Maintenance for a Markov Deteriorating System Under Scenario-based Demand Uncertainty

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ABSTRACT

In this paper, a single-product, single-machine system under Markovian deterioration of machine condition and demand uncertainty is studied. The objective is to find the optimal intervals for inspection and preventive maintenance activities in a condition-based maintenance planning with discrete monitoring framework. At first, a stochastic dynamic programming model whose state variable is the machine status is presented. In the first model, the demand is assumed to be deterministic and the objective is to minimize the sum of inspection, preventive maintenance, and lost sale costs. Then, in order to take the demand uncertainty into account, the extended model is formulated as a scenario-based two-stage stochastic programming one. In the second model, selecting the best inspection plan and finding the appropriate intervals for preventive maintenance are considered as the first and second stage decisions, respectively. Analyzing an illustrative example to study the effect of demand uncertainty in the problem shows that the total average cost is a non-decreasing function of machine state and demand. Moreover, if the machine state is worsened or the demand is increased, the number of inspections increase and the preventive maintenance should be executed at the same time or earlier. Finally, when the unit lost sale cost is greater than a certain amount, ignoring the demand uncertainty is not costly.

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1. INTRODUCTION

In recent decades, with a view toward just in time (JIT) in production and operation management, the production procedures have changed and JIT roles have become prominent. Machine deterioration is one of the main reasons for production capacity loss and consequently of delay in customer requests in many manufacturing industries. Growing machine health by preventive maintenance is a policy to restore production capacity, improving the timely delivery of customer requests [1]. However, preventive maintenance decreases machine unavailability and thus increases the potential production capacity of the machine for processing customer request. Therefore, adopting a preventive maintenance strategy that can keep the machine in a position to respond as much as possible to customer orders is a challenging problem [2].

Generally, in literature there are two strategies for preventive maintenance (PM), namely time-based maintenance (TBM) and condition-based maintenance (CBM). Traditionally, PM is carried out in the form of system overhaul or unit replacement based on the elapsed time, which is often mentioned to as time-based maintenance. TBM policies are usually approved based on a probabilistic model of system failure. In TBM, the machine age is the basis of the planning and the maintenance will be carried out after a specific period of time regardless of the health status of a physical asset and customer demand. In this approach, employing a suitable policy to determine PM durations and the frequencies is very important because an over frequent policy leads to additional cost and an over duration policy leads to unexpected failures [3, 4].

In recent years, to reduce the number of unnecessary scheduled preventive maintenance operations and

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eliminate the risks associated with them, more efficient maintenance approaches such as condition-based maintenance have emerged [5]. Unlike TBM policies built on historical failure data, CBM is a maintenance approach that emphasizes the information collected through condition monitoring. This policy consists of two stages. In the first stage, system status is evaluated and the machine state is identified. This stage can be either a continuous monitoring (inspection) or a discrete one. In continuous monitoring, which may be costly in some industries, some sensors continuously control the system status. By continuous monitoring, one continuously monitors (usually by mounted sensors) a machine and triggers a warning alarm whenever something wrong is detected. There are two limitations in the continuous monitoring. Firstly, it is often expensive and secondly, it produces inaccurate diagnostic information due to monitoring of raw signals with noise, continuously [6]. In discrete monitoring, some machine's significant covariates will be measured within a specific time period. Although this approach is more economical than the continuous one, there is always the risk of missing some alert between two inspections. In this type of monitoring, if the inspection is over frequent the inspection cost will increase, and in contrast, the cost of unnecessary PM operation will decrease. In contrast, if the inspection frequency decreases, the total cost of PM operation and unexpected failure will increase despite the decrease in inspection cost. Accordingly, there are different approaches to determine the duration and frequency of inspections, in discrete monitoring. The first one is to do it on a fixed interval. However, in the second approach, the duration is constant, but frequency achieves by running a model considering different economic measurements. In the third approach, the duration is not constant and the frequency is not predefined but by running a model the optimal plan will be extracted [7-10]. Furthermore, all possible inspection schemes were incorporated to prevent local optimum solution and avoid unnecessary combinations [11]. In fact, the possibility that there are savings in the number of inspections undertaken was considered [12]. As such, a proportional hazards model was used for risk of failure and a Markovian process to model the system covariates. In the second stage, based on the collected information from the first stage, the PM execution interval will be determined. In CBM, the PM operations will be executed if the machines significant covariates are more than predefined values called threshold. The main challenge in CBM is determined the optimum threshold policy for PM execution. Base on literature [13, 14], a maintenance policy for a degrading system with age- and state-dependent operating cost in which cost is increased with aging of the system and degradation levels was proposed. Ghandali et al. [15] have proposed a partially observable Markov decision process structure in which inspection and maintenance optimal strategies must be adapted to maximize the system availability and the expected value of profit together.

In the optimization procedure of the previous models, customer requirements have been neglected and the modeling was based on the hazard rate function of the machine. In the following, some models are investigated in which production and PM planning have been integrated. The problem of production and maintenance planning of a multiple-product manufacturing system with a single deteriorating machine was modeled as a Markov decision process [16, 17]. The objective was to choose simultaneously the equipment maintenance plan as well as the quantity to produce in a way that the sum of expected production, backorder, and holding costs were minimized. A semi-Markov decision process model for a single-stage production system with multiple products and multiple maintenance actions was presented by Sloan, Kang and Subramaniam [18, 19]. Other factors such as the lost sale cost added to previous factors to run the optimization model for threshold [2, 20, 21]. In none of these models, inspection has not been a decision variable, and the cost of inspection has not been taken into account, although other decisions in production planning such as production, inventory, and backorder quantity have been considered.

To the best of our knowledge, so far there has been no model in which inspection and preventive maintenance planning are simultaneously determined in the presence of the Markovian deteriorating machine conditions and under scenario-based uncertain demand. However, in practice, demand motivates running the machine. For example, consider a situation in which the optimum time of PM execution coincides with the customer order reparation and delivery. This coincidence leads to a delay in order delivery and accordingly customer dissatisfaction. Therefore, delay in PM operations result in unexpected failure and also have an undesired effect on the lost sale cost and customer's satisfaction, especially in uncertain situations. The key contribution of this paper is summarized in the following:

We have taken scenario-based uncertain demand into account and analyzing the effect of it on decisions and costs when it is decided about the inspection and the PM operations, simultaneously. To achieve this goal, employing stochastic dynamic programming, two models for condition-based maintenance planning in tactical level and finite time horizon are presented. In both models, the discrete inspections are considered that its frequencies are neither predefined nor fixed, but they are decision variables. Moreover, based on the result of the inspection, the optimum interval for PM, which depends on the state variable whose value is emerged during the inspection stage, will be determined. In the first model, demand is certain but the machine status is not certain. This uncertainty is defined by considering the machine status as a state variable, and the related decision variables will be calculated such that the sum of inspection, PM, and lost sale costs are minimized. The second model is developed by assuming uncertainty for the demand and by a scenario-based two-stage stochastic programming approach. Each scenario is a stochastic vector whose elements show the demand in their corresponding periods. Considering the set of scenarios as a sample space of a random experiment, we deal with a probability distribution function (p.d.f). In the second model, the total cost of two successive inspections, as a function of the scenario, is another random variable and its conditional expected value given a specific scenario is placed in the optimality equation. This new arrangement of the optimality equation allows the uncertainty of the demand to be taken into account substantially. We also run these models separately for all possible inspection plans (2^K possibilities, where k equals the number of time periods) and compare the results and find the optimal combination of inspection and PM time. Furthermore, in order to analyze the effect of demand uncertainty on total cost, two measurements known as value of stochastic solution (VSS) and the expected value of perfect information (EVPI), which have been defined in literature [22], are calculated.

This paper is prepared as follows. Section 2 explains the problem and assumptions associated with it in general. Section 3 displays our modeling framework including appropriate mathematical representation. Section 4 gives a solution algorithm. Numerical study and its computational sequences have been described in Section 5. Finally, Section 6; concludes the paper.

2. PROBLEM STATEMENT

First of all, a single product system whose production is based on the customer order is considered and the planning is done in a finite time horizon with K equal intervals with length of T. Let Δ, a discrete random vector of K elements whose probability distribution function (p.d.f) is g_Δ, be the demand in the planning horizon. The range of Δ is shown by R_Δ each member of it represented by $\delta = (\delta(1), \ldots, \delta(K))$ is a demand scenario, whose elements show the demand in different periods, determined at the beginning of the period. The demand of every period must be satisfied in that period. If the system cannot respond to the demand completely, for every unit of the unsatisfied demand, a value represented by h will be lost. For simplicity, the whole production system is considered as a single machine which deteriorates during its operation time because of production. A variety of levels are defined for the machine deterioration, called state, and the set of machine states are shown as $S = \{0,1,2,...,N\}$. The deterioration is assumed to be a Markov stochastic process, i.e. if $X = (X_t : t \in [0,\infty))$ be the machine deterioration process and assume X to be a homogeneous continuous time Markov process with discrete state space S, then the deteriorating machine can be in one of N operational states $0,1,2,...,N$ -1 or in a failure state N. The machine has the best performance in the state 0 and is out of service in the state N. In other states, a larger number shows a lower performance, hence $r(s)$ is an absolutely decreasing function while $r(s)$ is the machine performance in the state s.

As mentioned before, two models are proposed in the stochastic dynamic programming form for overcoming this problem. Both models consist of two stages: the first stage is related to inspections while the second one includes decision making about appropriate interval for PM activities.

In the first stage, it is assumed that in the inspection the machine state is observable, i.e. its specification is error-free (in contrast with the hidden state). Moreover, it is assumed that at the beginning of the planning horizon, the inspection must be executed. The successive inspection will be carried out only at the beginning of each period while conducting the inspections depends on our decision. The cost and inspection processing time are constant parameters and will be shown by c_{ins} and t_{ins}, respectively. In the second stage, it is assumed that the machine state becomes zero just after the execution of PM operation. PM cost and its processing time which depend on machine state (s) are absolutely increasing function in s. These parameters are shown by $c_{pm}(s)$ and $t_{pm}(s)$, respectively. Moreover, we assume that $c_{pm}(0)$, $t_{pm}(0)$ are equal to zero.

The inspection plan identifies the inspection frequency and therefore has a direct impact on inspection cost. However, it has an indirect impact on other costs because other procedures will be carried out after inspection execution and they depend on the inspection plan. In addition to inspection cost, there are two types of costs. The first type is the PM cost and the second type is the lost sale cost which is due to machine unavailability during inspection and/or PM operations. In order to consider this cost, besides creating a dependency between production rate and machine state, the time elapsed for inspection and PM operations is also taken into account. This cost is proportional to the difference between actual production capacity and certain demand in the first model and the expected value of the difference between actual production capacity and stochastic demand in the second model.

The goal is to find the optimal intervals for inspection and PM activities in a condition-based maintenance planning with discrete monitoring (CBMDM) framework over a finite time horizon to satisfy the scenario-based uncertain demand. Here, the appropriate PM interval must be based on the inspection results so that the total cost in planning horizon, including inspection, PM, and lost sale cost, is minimized.

3. THE MATHEMATICAL MODEL

To develop the model, in this section, initially the basic model of finite time horizon stochastic dynamic programming is explained and the first model with certain demand is made. Then, the second model is extended to take the scenario-based uncertain demand into account.

3. 1. Basic Model of Stochastic Dynamic Programming with Finite Time Horizon

In this research, we use stochastic dynamic programming with finite time horizon method for modeling the problem. In this method, deriving the optimality equation that is a backward recursive equation is the most important part of the modeling. The components of this equation are the stage (n), state variable (s), action (a), transition probability matrix ($P(a)$) and current stage cost $(C(s,a))$ when the system state is s and action a is decided. Equation (1) shows the optimality equation in the general form:

$$V_n(s) = \min_{a \in \mathrm{A}} \{C(s,a) + \sum_{s' \in \mathrm{S}} P_{ss'}(a) V_{n-1}(s')\} \quad (\forall n \geq 1) \tag{1}$$

where, S, A are state and action spaces, respectively, and $V_n(s)$ shows expected total cost when the system state is s and there are n stages to the end of the planning horizon. Moreover, it is assumed that $V_0(s) = 0 \ (\forall s \in \mathrm{S})$ [23].

3. 2. Proposed Model I with Certain Demand

In this section, the problem is modeled in a stochastic dynamic programming form without considering the demand uncertainty. For this purpose, first, the optimality equation components are defined and, then the optimality equation is made.

3. 2. 1. Components of the Proposed Model I

I. The First Component (Stage) Each point of decision about inspection is defined as a stage. The structure of model I is such that it requires in each stage the number of the remained periods to the end of the planning horizon. The beginning of each period is an option for inspection. Therefore, the inspection in each period is considered as a binary decision variable. Each feasible solution is an array consists of 0 and 1s which is shown by $I = (i_k)_{k=0}^{K}$ and referred to an ***inspection plan***. It is assumed that an inspection is performed at the beginning of the first period. Also, for simplicity, we assume that the last element of each sequence is 1. That is, each sequence starts with and ends to 1, where the 1 at the end of the sequence is not a real inspection and is only for the simplicity of modeling, i.e.

$$I = (i_0, i_1, \ldots, i_K) = (i_k)_{k=0}^{K} \qquad , i_0 = i_K = 1$$

Where, K is the number of periods included in the planning horizon, the set of feasible solutions is shown by I. Therefore, the number of elements of I is equal to 2^{K-1}.

Assume that sequence $I = (i_k)_{k=0}^{K}$ is an arbitrary inspection plan and henceforth constant. Assume that $\bar{I}$ is a subsequence of I that consists of its 1s, i.e.

$$\bar{I} = (\bar{i}_{k_0}, \bar{i}_{k_1}, \cdots, \bar{i}_{k_{m(I)}}) = (\bar{i}_{k_i})_{i=0}^{m(I)}$$

so that,

$$\bar{i}_{k_i} = 1 \quad \forall i \in \{0, 1, \cdots, m(I)\}$$

and

$$i_k = 0 \quad \forall k \notin \{k_0, k_1, \cdots, k_{m(I)}\}$$

Now, the sequence $J(I) = (j(I)_n)_{n=0}^{m(I)}$ is defined as follows:

$$J(I) = (j(I)_0, j(I)_1, \cdots, j(I)_{m(I)}) =$$
$$(K - k_{m(I)}, K - k_{m(I)-1}, \cdots, K - k_1, K - k_0)$$

The sequence $J(I)$ that indirectly identifies the stages is named as ***stages sequence***. The number of sentences in this sequence determines the number of stages.

Example 1. Assume that $I = (1,0,0,1,0,1,1)$ is an inspection plan for a problem with six periods. As shown in Figure 1, in this inspection plan, inspections are carried out at the beginning of the first, fourth, and sixth periods.

According to the definition, the last sentence in the sequence I is always considered 1, therefore, it is shown differently from the rest. Consider the subsequence $\bar{I} = (1,1,1,1)$ that consists of ones of the sequence I and the sequence $k = (0,3,5,6)$ which contains the index of these ones.

The stages sequence corresponded to I is obtained as follows:

$$J(I) = (6-6, 6-5, 6-3, 6-0) = (0,1,3,6)$$

This sequence shows that for the inspection plan $I = (1,0,0,1,0,1,1)$, the stochastic dynamic programming model includes four stages: first, second, third, and fourth stages show times when zero, one, three, and six time periods are remained until the end of the planning horizon, respectively.

II. The Second Component (State Variable) The machine status is defined as the state variable of the model, and the indexes s and s' are considered for it, so that $s, s' \in \mathrm{S}$.

III. The Third Component (Decision Variable) In our problem, there are two types of decision variables. The first type specifies the periods in which the machine should be inspected and the second one specifies, at each stage until the next inspection time (the next stage), the period selected for the PM execution according to the state (machine status) of the current stage- Note that there

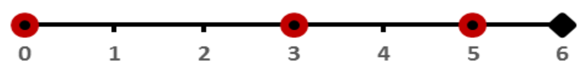

Figure 1. Inspection scheme for the example 1

is the possibility of not executing the PM in a period. Each inspection plan is a finite sequence of the first type decision variables. Assume that $I=(i_k)_{k=0}^{K}$ is an arbitrary inspection plan and henceforth is constant and $J(I)=(j(I)_n)_{n=0}^{m(I)}$ is its corresponding stages sequence in the dynamic stochastic programming model. In the stage *n*, the second type decisions' space correspond to this inspection plan is $\mathrm{A}=\{0,1,...,l_{I,n}\}$, such that $l_{I,n}=j(I)_n-j(I)_{n-1}$. In this space, decision 0 means that after the implementation of the inspection, the decision to perform preventive maintenance is not made, and decision *a* means that the preventive maintenance will be implemented during the a^{th} period between the current and the next inspections $(1\le a\le l_{I,n})$.

IV. Fourth Component (Transition Probability Matrix) As mentioned before, we assume that the machine deterioration process is a homogeneous continuous time Markov process with a discrete state space $\mathrm{S}=\{0,1,\dots,N\}$ shown by $\mathrm{X}=(X_t:t\in[0,\infty))$. Having the transition rate matrix is sufficient for attaining the transition probability matrix of the process. The transition rate matrix *Q* is defined as follows:

$$Q=[q_{ss'}]_{(N+1)\times(N+1)}$$

$$q_{ss'}=\lim_{t\to 0}\frac{Pr(X_t=s'\mid X_0=s)}{t},\ \ s,s'\in\mathrm{S},\ \ s\ne s',$$

$$q_{ss}=-\sum_{s'\ne s}q_{ss'}.$$

Furthermore, we assume that the following conditions hold on the matrix *Q*:

Condition1: Without implementing PM, the machine status deteriorates because of production, i.e.

$$q_{ss'}=0,\ \forall s'<s \tag{2}$$

Condition2: The rate of transition to inferior states increments, as a result of deteriorating of the machine. In other words,

$$\sum_{s'\ge u}q_{ss'}<\sum_{s'\ge u}q_{(s+1)s'},\ \forall u\in\mathrm{S},u\ge(s+2) \tag{3}$$

Now, the transition probability matrix of the process is obtained by using the Chapman-Kolmogorov equation [23]. Hence, assume that P_0 and P_1 are these matrices after elapsing a time interval with length *T* when PM is not carried out and when PM is carried out, respectively. So we have:

$$P_0=e^{\mathrm{Q}T} \tag{4}$$

$$P_1=R\times P_0=R\times e^{\mathrm{Q}T} \tag{5}$$

where, *R* is defined as follows:

$$R=[r_{ss'}]_{(N+1)\times(N+1)}$$

$$r_{ss'}=\Pr(X_{0^+}=s'\mid X_0=s,a=1)$$

But because we assume that the machine after performing PM is restored to an as-good-as-new status, we have:

$$R=\begin{bmatrix}1 & 0 & \cdots & 0\\ \vdots & 0 & \cdots & 0\\ 1 & 0 & \cdots & 0\end{bmatrix}$$

V. Fifth Component (Total Cost Between Two Successive Inspections) Assume that $C_{n,l}^{Z}(s,a)$ is the total cost between the current inspection and the next one (two successive inspections) in which *Z*, *n*, *s*, *l*, and *a* are demand vector, current inspection index in an inspection plan, machine status in the inspection, the number of time periods between the current inspection and the next one, and a member of decision space $\mathrm{A}=\{0,1,...,l\}$, respectively. $C_{n,l}^{Z}(s,a)$ is calculated using the Equations (6)-(8). In these equations, ρ is a discount factor and $z(w)$ is the demand in the period *w*. $T_{A1}(s,b)$ and $T_{A2}(s,b)$ are actual time for production operations in one period in the case of performing and not performing the inspection, respectively, such that the machine is in the state *s* at the beginning of the time period and action *b* is decided for PM execution. These expressions are calculated as follows:

$$T_{A1}(s,b)=T-t_{ins}-bt_{pm}(s),\quad b\in\{0,1\}$$

$$T_{A2}(s,b)=T-bt_{pm}(s),\quad b\in\{0,1\}$$

The $r(s,b)$ is the machine production rate in one period such that the machine is in the state *s* at the beginning of the time period and action *b* is decided for PM execution:

$$r(s,b)=\begin{cases}r(s) & b=0\\ r(0) & b=1\end{cases}$$

The β_h is an indicator function such that:

$$\beta_h:\{h,\cdots,K\}\to\{0,1\}$$

$$\beta_h(l)=\begin{cases}0 & l=h\\ 1 & l\ge h+1\end{cases}$$

Equation (6) calculates the cost between two successive inspections in a situation where the current inspection index is *n*, machine status in the inspection is *s*, the number of time periods until the next inspection is *l*, and PM action is not implemented between these two inspections. The terms of this equation are explained in Table 1.

TABLE 1. The explanation of the Equation (6)

c_{ins} Inspection cost
$h[z(n)-T_{A1}(s,0)r(s,0)]^{+}$ Lost sale cost for the first period between two successive inspections
$\rho^{w}\sum_{s'=0}^{N}(P_0^{w})_{s,s'}h[z(n+w)-T_{A2}(s',0)r(s',0)]^{+}$ The discounted expected value of the lost sale cost for the $(w+1)^{\text{th}}$ period between two successive inspections ($w\in\{1,\dots,l-1\}$)

$$C^{Z}_{n,l}(s,0)=c_{ins}+h[z(n)-T_{A1}(s,0)r(s,0)]^{+}+\beta_1(l)$$
$$\sum_{w=1}^{l-1}\rho^{w}\sum_{s'=0}^{N}(P_0^{w})_{s,s'}h[z(n+w)-T_{A2}(s',0)r(s',0)]^{+} \qquad (6)$$
$$(n\in\{1,...,K\},\ l\in\{1,\cdots,K-n+1\},s\in \mathrm{S})$$

Equation (7) calculates this cost in similar conditions with the difference that PM action is implemented in the first time period between those two inspections. The expressions of this equation are described in Table 2.

$$C^{Z}_{n,l}(s,1)=c_{ins}+c_{pm}(s)+h[z(n)-T_{A1}(s,1)r(s,1)]^{+}$$
$$+\beta_1(l)\sum_{w=1}^{l-1}$$
$$\rho^{w}\sum_{s'=0}^{N}(P_1P_0^{w-1})_{s,s'}h[z(n+w)-T_{A2}(s',0)r(s',0)]^{+} \qquad (7)$$
$$(n\in\{1,...,K\},\ l\in\{1,\cdots,K-n+1\},s\in \mathrm{S})$$

If the number of time periods between two successive inspections is greater than one, then Equation (8) is used for calculating $C^{Z}_{n,l}(s,a)$. The parts of this equation are disclosed in Table 3.

$$C^{Z}_{n,l}(s,a)=c_{ins}+h(z(n)-T_{A1}(s,0)r(s,0))^{+}$$
$$+\beta_2(a)\sum_{w=1}^{a-2}\rho^{w}\sum_{s'=0}^{N}(P_0^{w})_{s,s'}h[z(n+w)$$
$$-T_{A2}(s',0)r(s',0)]^{+}$$
$$+\rho^{a-1}\sum_{s'=0}^{N}(P_0^{a-1})_{s,s'}\{c_{pm}(s')+h[z(n+a-1)$$
$$-T_{A2}(s',1)r(s',1)]^{+}\} \qquad (8)$$
$$+\beta_a(l)\sum_{w=a}^{l-1}\rho^{w}\sum_{k=0}^{N}(P_0^{a-1}P_1P_0^{w-a})_{s,s'}h[z(n+w)$$
$$-T_{A2}(s',0)r(s',0)]^{+}$$
$$(a\in\{2,\cdots,K\},n\in\{1,...,K-a+1\},$$
$$l\in\{a,\cdots,K-n+1\},s\in \mathrm{S})$$

The role of the indicator function β_h in Equations (6)-(8) is that, under some conditions, a statement of the equation may be omitted, for example in Equation (6), if the distance between two successive inspections is one period, Equation (6) follows that*:*

TABLE 2. The explanation of the Equation (7)

c_{ins} Inspection cost
$c_{pm}(s)$ Preventive maintenance cost
$h[z(n)-T_{A1}(s,1)r(s,1)]^{+}$ Lost sale cost for the first period between two successive inspections
$\rho^{w}\sum_{s'=0}^{N}(P_1P_0^{w-1})_{s,s'}h[z(n+w)-T_{A2}(s',0)r(s',0)]^{+}$ The discounted expected value of the lost sale cost for the (w+1)th period between two successive inspections $(w\in\{1,\ldots,l-1\})$

TABLE 3. The explanation of Equation (8)

c_{ins} Inspection cost
$h(z(n)-T_{A1}(s,0)r(s,0))^{+}$ Lost sale cost for the first period between two successive inspections
$\rho^{w}\sum_{s'=0}^{N}(P_0^{w})_{s,s'}h[z(n+w)-T_{A2}(s',0)r(s',0)]^{+}$ The discounted expected value of the lost sale cost for the (w+1)th period between two successive inspections before PM execution $(w\in\{1,\ldots,a-2\})$
$\rho^{a-1}\sum_{s'=0}^{N}(P_0^{a-1})_{s,s'}\{c_{pm}(s')+h[z(n+a-1)$ $-T_{A2}(s',1)r(s',1)]^{+}\}$ The discounted expected value of the sum of PM and lost sale costs for the (a)th period between two successive inspections
$\rho^{w}\sum_{k=0}^{N}(P_0^{a-1}P_1P_0^{w-a})_{s,s'}h[z(n+w)$ $-T_{A2}(s',0)r(s',0)]^{+}$ The discounted expected value of the lost sale cost for the (w+1)th period between two successive inspections after PM execution $(w\in\{a,\ldots,l-1\})$

$$C^{Z}_{n,l}(s,0)=c_{ins}+h[z(n)-T_{A1}(s,0)r(s,0)]^{+}$$

In Equations (6)-(8), the domain of each index is such that it contains all the combinations that may arise in the calculation of the cost between two successive inspections. Example 2 is designed for this purpose.

Example 2. Suppose that in a problem K = 6, then the boundaries of Equations (6)-(8) are defined as shown in Table 4.

Furthermore, In the calculation of the optimal solution of a problem with K=6 by using the proposed algorithm in section 4, depending on the considered inspection scheme, some of these seventy-seven combinations will be used to calculate $C^{Z}_{n,l}(s,a)$.

3. 2. 2. Optimality Equation of the Proposed Model I

In this section, respecting the definition of the optimality equation's components mentioned in the previous section, an optimality equation is constructed for each inspection plan. For this purpose, suppose that $I=(i_k)_{k=0}^{K}$ is an arbitrary inspection plan and henceforth constant and $J(I)=(j(I)_n)_{n=0}^{m(I)}$ is its corresponding stages sequence. Then, Equation (9) shows the optimality equation of the model I.

$$V_{j(I)_n}(s)=\min_{a\in\{0,1,\ldots,l_{I,n}\}}\{C^{Z}_{j_n,l_{I,n}}(s,a)+$$
$$\rho^{l_{I,n}}\sum_{s'=0}^{N}(\mathrm{P}(a))_{s,s'}V_{j(I)_{n-1}}(s')\} \qquad (9)$$
$$n\in\{1,\ldots,m(I)\}$$

where,

TABLE 4. Total boundaries of Equations (6)-(8)

	a	*n*	*l*	Number of combinations
boundaries of Equation (6)	0	1	$l \in \{1,2,3,4,5,6\}$	6
		2	$l \in \{1,2,3,4,5\}$	5
		3	$l \in \{1,2,3,4\}$	4
		4	$l \in \{1,2,3\}$	3
		5	$l \in \{1,2\}$	2
		6	$l \in \{1\}$	1
Number of combinations in the Equation (6)				**21**
boundaries of Equation (7)	1	1	$l \in \{1,2,3,4,5,6\}$	6
		2	$l \in \{1,2,3,4,5\}$	5
		3	$l \in \{1,2,3,4\}$	4
		4	$l \in \{1,2,3\}$	3
		5	$l \in \{1,2\}$	2
		6	$l \in \{1\}$	1
Number of combinations in the Equation (7)				**21**
boundaries of Equation (8)	2	1	$l \in \{2,3,4,5,6\}$	5
		2	$l \in \{2,3,4,5\}$	4
		3	$l \in \{2,3,4\}$	3
		4	$l \in \{2,3\}$	2
		5	$l \in \{2\}$	1
	3	1	$l \in \{3,4,5,6\}$	4
		2	$l \in \{3,4,5\}$	3
		3	$l \in \{3,4\}$	2
		4	$l \in \{3\}$	1
	4	1	$l \in \{4,5,6\}$	3
		2	$l \in \{4,5\}$	2
		3	$l \in \{4\}$	1
	5	1	$l \in \{5,6\}$	2
		2	$l \in \{5\}$	1
	6	1	$l \in \{6\}$	1
Number of combinations in Equation (8)				**35**
Number of total combinations in Equations (6)-(8)				**77**

$V_{j(I)_0}(s) = 0 \ \ (s \in S)$, $\bar{j}_n = K - j(I)_n + 1$,

$l_{I,n} = j(I)_n - j(I)_{n-1}$,

$$\mathrm{P}(a) = \begin{cases} P_0^{\,l_{I,n}} & a = 0 \\ P_0^{\,a-1} P_1 P_0^{\,l_{I,n}-a} & a \in \{1,2,\ldots,l_{I,n}\} \end{cases}.$$

3. 3. Proposed Model II with Uncertain Demand

In this section, to consider the uncertainty of the demand, the first model is extended using the uncertain demand Δ instead of *Z*. The model is formulated as a stochastic dynamic programming model, the same as the first model. The components of the optimality equation for model II are defined as for model I except the state variable and the cost between two successive inspections. The state variable is changed to (s,z). In order to calculate the cost, $\bar{C}_{n,l}(s,z,a) = E[C^{\Delta}_{n,l}(s,a) \mid A_n(z)]$ should be replaced with $C^{Z}_{n,l}(s,a)$, in which $A_n(z)$ is the event of all scenarios whose n^{th} component is z and $E[\cdot \mid \cdot]$ represents the conditional expected value. Therefore, in the extended model, the cost between two successive inspections is obtained as Equation (10).

$$\begin{aligned} \bar{C}_{n,l}(s,z,a) &= E[C^{\Delta}_{n,l}(s,a) \mid A_n(z)] \\ &= \sum_{\delta \in \mathrm{R}_\Delta} C^{\delta}_{n,l}(s,a) P(\Delta = \delta \mid A_n(z)) \\ &= \sum_{\delta \in A_n(z)} C^{\delta}_{n,l}(s,a) \frac{g_\Delta(\delta)}{P(A_n(z))} \\ &= \frac{1}{P(A_n(z))} \sum_{\delta \in A_n(z)} C^{\delta}_{n,l}(s,a) g_\Delta(\delta) \\ & s \in \mathrm{S}, \delta \in \mathrm{R}_\Delta, \\ & (a \in \{0\}, n \in \{1,\cdots,K\}, l \in \{1,\cdots,K-n+1\}) \ or \\ & (a \in \{1,\ldots K\}, n \in \{1,\cdots,K-a+1\}, l \in \{a,\cdots,K-n+1\}) \end{aligned} \tag{10}$$

where, $P(A_n(z)) = \sum_{\delta \in A_n(z)} g_\Delta(\delta)$.

Now, the optimality equation of the model II is obtained by adjusting Equation (9) as shown in Equation (11).

$$\begin{aligned} V_{j(I)_n}(s,z) = \min_{a \in \{0,1,\ldots,l_{I,n}\}} \{ & \bar{C}_{\bar{j}_n, l_{I,n}}(s,z,a) + \rho^{l_{I,n}} \\ & \sum_{\alpha \in \mathrm{R}_\Delta(\bar{j}_{n-1})} \sum_{s'=0}^{N} (P(A_{\bar{j}_{n-1}}(\alpha)) \mathrm{P}(a))_{s,s'} V_{j(I)_{n-1}}(s',\alpha) \} \\ & (n = 1,\ldots,m(I)) \end{aligned} \tag{11}$$

where,

$V_{j(I)_0}(s,z) = 0$, $(\forall \mathrm{s} \in \mathrm{S},\ z \in \mathrm{R}_\Delta)$,

and,

$\mathrm{R}_\Delta(\bar{j}_{n-1})$ is a set of all $\bar{j}_{n-1}$ whose n^{th} components belongs to R_Δ. $\bar{j}_n$, $l_{I,n}$ and $\mathrm{P}(a)$ are equal to those considered for the Equation (9).

4. SOLUTION METHOD

In order to solve the models presented in the previous section, the following four-step algorithm is used. For simplicity in describing the algorithm, the symbol ζ is used for the state variable in both models. That is, if the following algorithm is used to solve the first model, then $\zeta = s$, and if it is used to solve the second model, then $\zeta = (s, z)$.

Step 1: For each element of $\mathbf{I}$ as $I = (i_k)_{k=0}^{K}$ (i.e. an arbitrary inspection plan and henceforth constant) execute steps 2 and 3.

Step 2: Make the stages sequence $J(I) = (j(I)_n)_{n=0}^{m(I)}$ corresponding to the inspection plan $I = (i_k)_{k=0}^{K}$.

Step 3: For each state ζ **,** using the optimality equation, that is a backward recursive equation, calculate the optimal value and find the corresponding optimal decision and then name them $V^*_{j(I)_{m(I)}}(\zeta)$ and $a_I{}^*(\zeta)$, respectively. In other words, $a_I{}^*(\zeta)$ is the optimal decision about the time of executing of PM actions in the interval between the first and the second inspections when inspection plan is I and system state at the beginning of time horizon is ζ .

Step 4: For each state. ζ **.** select the inspection plan with the minimum value $V^*_{j(I)_{m(I)}}(\zeta)$ as the optimal inspection plan corresponding to ζ in planning horizon and name it as $I^* = (i_k^*)_{k=0}^{K}$.

Step 5: For each state of ζ and for the optimal inspection plan I^* corresponding to ζ , select $a_{I^*}{}^*(\zeta)$ as an optimal decision for PM execution between the first and second inspections.

5. NUMERICAL STUDY

In this section, an illustrative example is designed to analyze the strategy of the proposed models as well as investigate the effect of demand uncertainty in problem modeling.

5. 1. Detailed Example Suppose that the planning horizon consists of 6 time periods each of which is 30 days along, the machine states space is $\mathrm{S} = \{0,1,2,3,4\}$, the cost of each unit of lost sale is \$5, the cost of each inspection is \$250, the execution time of each inspection is one day and the discount factor is one. Relationship (12) shows the transition rate matrix that applies to relations (2) and (3) in order to satisfy the conditions 1 and 2. The one-step transition matrixes are calculated in the absence and in the presence of PM execution by relations (4) and (5), respectively and shown in relations (13) and (14). Other parameters are listed in Table 5.

$$\mathrm{Q} = \begin{bmatrix} -0.0241 & 0.0134 & 0.0086 & 0.0011 & 0.0010 \\ 0 & -0.0200 & 0.0105 & 0.0051 & 0.0044 \\ 0 & 0 & -0.0144 & 0.0144 & 0 \\ 0 & 0 & 0 & -0.0133 & 0.0133 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$P_0 = \begin{bmatrix} 0.485 & 0.208 & 0.180 & 0.075 & 0.052 \\ 0 & 0.545 & 0.188 & 0.137 & 0.130 \\ 0 & 0 & 0.650 & 0.284 & 0.066 \\ 0 & 0 & 0 & 0.670 & 0.330 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$P_1 = \begin{bmatrix} 0.485 & 0.208 & 0.180 & 0.075 & 0.052 \\ 0.485 & 0.208 & 0.180 & 0.075 & 0.052 \\ 0.485 & 0.208 & 0.180 & 0.075 & 0.052 \\ 0.485 & 0.208 & 0.180 & 0.075 & 0.052 \\ 0.485 & 0.208 & 0..180 & 0.075 & 0.052 \end{bmatrix} \tag{14}$$

In order to investigate the demand uncertainty, 32 different scenarios for demand in different periods are generated, each of which has a probability of occurrence equal to $1/32$, and their tree is shown in Figure 2. The demand in the first period is 300, in the second and the third periods is 20% more or less than the previous one, and from the fourth period thereafter is 10% more or less than the previous one.

5. 2. Results Analysis MATLAB software is used to implement the solution algorithm. The goal of solving both models is to find the optimal inspection plan (the first stage decisions) in the planning horizon, as well as the appropriate interval to execute the PM (the second stage decisions) according to the state of the system at the inspection time. In the first model, the demand is certain and system state refers only to the machine status. While in the second model, the demand is uncertain and scenario-based and the state of the system shows both the status of the machine and the demand at the inspection

TABLE 5. Cost and executing time of PM and production rate for each machine state

s	$c_{pm}(s)$	$t_{pm}(s)$	$r(s)$
0	0	0	20
1	300	1	16
2	500	2	10
3	900	3	2
4	1500	4	0

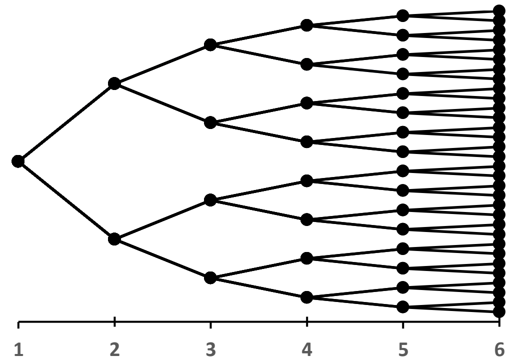

Figure 2. Scenarios tree

time. Table 6 shows the optimal inspection plans for the first and second models. These results indicate that the optimal inspection plan in the first model depends on the scenario and machine status at the beginning of the planning horizon, especially, higher demand or worse machine status will increase the number of inspections. Also, the optimal inspection plan for the second model depends, in addition to the machine status, on the demand at the beginning of the planning horizon. For example, the inspection plan I_5, which is the optimal solution of the first model for the average scenario and states 1 and 2, is not optimal for states $(1,300)$ and $(2,300)$ in the second model and instead, I_{11} is optimal inspection plan.

Tables 7 and 8 show the optimal solution to execute the PM according to the state of the system at the inspection time corresponding to the first and second model, respectively. Note that in the second model, the optimal PM execution interval depends on the demand and the scenario that has occurred up to that moment. In Table 8, each number that refers to demand represents the set of all scenarios whose demand in that specific period is equal to the same demand quantity. Therefore, decisions about the PM execution interval for scenarios with the same demand should be alike. Moreover, if the machine state is worsened or the demand is increased, the preventive maintenance should be executed at the same time or earlier.

5. 3. Discussion and Sensitivity Analysis

In the previous section, the numerical study showed how models work and how their solutions are utilized. Now, in this section, we intend to compare the performance of the two models for the same numerical example. Also, we investigate amount of reduction occurs in the total cost considering the demand uncertainty in the model. For this purpose, initially, the first model without considering demand is solved. For this purpose, in each period, the nominal production capacity $(600 = 30 \times 20)$ is replaced with its demand. The scenario, which generated with this method, is called the ***nominal capacity*** scenario. Then the selfsame model is solved for ***optimistic***, ***average***, and ***pessimistic*** scenarios separately, that these are new scenarios with the least, average, and highest demand in each period, respectively. Afterward, the performance of the optimal inspection plan (first stage decision) obtained from the first model for aforementioned scenarios are evaluated. For example, for the optimistic scenario, the optimal value of the first model for the optimal inspection plan ($V^*_{j(I)_{m(I)}}(s)$) is calculated in case of occurrence of thirty-two main scenarios, and then its expected value, named $E_{optimistic}$, is defined as the performance of the inspection plan corresponding to the optimistic scenario. The same procedure is repeated for the nominal capacity, average and pessimistic scenarios, whose performance of their inspection plans are shown as $E_{nominal}$, $E_{average}$ and $E_{pessimistic}$, respectively. Then, the second model, which is capable of considering all the scenarios simultaneously, is solved. The solution of this model is named the stochastic solution or here and now (HN) and consists of an optimal inspection plan, which is satisfactory for all scenarios. Finally, this solution is compared with the performance of inspection plans corresponding to aforesaid scenarios. Table 9 shows this comparison. These results show that despite machine state at the beginning of the planning horizon, the stochastic solution has a better or the same result as the nominal capacity, optimistic, average, and pessimistic scenarios solution.

To analyze the sensitivity of solutions toward the unit lost sale cost *h*, VSS and EVPI measures in terms of percentage, are calculated for the different machine states

TABLE 6. Optimal inspection plans

Model I		
Scenario	***s***	**Optimal inspection plan**
Nominal capacity $(600, 600, 600, 600, 600, 600)$	0	$I_{11} = (1,0,1,0,1,0,1)$
	1	$I_{23} = (1,1,0,1,1,0,1)$
	2	
	3	
	4	
Optimistic (Least demand) $(300, 240, 192, 173, 156, 140)$	0	$I_1 = (1,0,0,0,0,0,1)$
	1	
	2	
	3	$I_{17} = (1,1,0,0,0,0,1)$
	4	
Average (Mean demand) $(300, 300, 300, 300, 300, 300)$	0	$I_5 = (1,0,0,1,0,0,1)$
	1	
	2	
	3	$I_{21} = (1,1,0,1,0,0,1)$
	4	
Pessimistic (Highest demand) $(300, 360, 432, 475, 523, 575)$	0	$I_{11} = (1,0,1,0,1,0,1)$
	1	
	2	
	3	$I_{23} = (1,1,0,1,1,0,1)$
	4	
Model II		
System State		**Optimal inspection plan**
$(0,300)$		$I_5 = (1,0,0,1,0,0,1)$
$(1,300)$		$I_{11} = (1,0,1,0,1,0,1)$
$(2,300)$		
$(3,300)$		$I_{21} = (1,1,0,1,0,0,1)$
$(4,300)$		

TABLE 7. Optimal PM execution interval obtained from the first model corresponding to the four scenarios

Nominal capacity

s	$a^*_{I_{11}=(1,0,1,0,1,0,1)}(s)$			$a^*_{I_{23}=(1,1,0,1,1,0,1)}(s)$			
	First inspection	**Second inspection**	**Third inspection**	**First inspection**	**Second inspection**	**Third inspection**	**Fourth inspection**
0	2	2	2		2	0	2
1		2	1	1	2	1	2
2		1	1	1	2	1	1
3		1	1	1	1	1	1
4		1	1	1	1	1	1

Optimistic scenario

s	$a^*_{I_1=(1,0,0,0,0,0,1)}(s)$	$a^*_{I_{17}=(1,1,0,0,0,0,1)}(s)$	
	First inspection	**First inspection**	**Second inspection**
0	3		2
1	2		2
2	2		2
3		1	1
4		1	1

Average scenario

s	$a^*_{I_5=(1,0,0,1,0,0,1)}(s,z)$		$a^*_{I_{21}=(1,1,0,1,0,0,1)}(s)$		
	First inspection	**Second inspection**	**First inspection**	**Second inspection**	**Third inspection**
0	3	2		2	2
1	2	2		2	2
2	2	2		2	2
3		1	1	1	1
4		1	1	1	1

Pessimistic scenario

s	$a^*_{I_{11}=(1,0,1,0,1,0,1)}(s)$			$a^*_{I_{23}=(1,1,0,1,1,0,1)}(s)$			
	First inspection	**Second inspection**	**Third inspection**	**First inspection**	**Second inspection**	**Third inspection**	**Fourth inspection**
0	2	2	2		2	0	2
1	2	2	2		2	1	2
2	2	1	1		2	1	1
3		1	1	1	1	1	1
4		1	1	1	1	1	1

at the beginning of the planning horizon, and with the same data as the previous section but a different value of h, using Equations (15) and (16) and then presented in Figures 3 and 4.

$$VSS\,(\%) = \frac{VSS}{E_{average}} = \frac{E_{average} - HN}{E_{average}} \tag{15}$$

$$EVPI\,(\%) = \frac{EVPI}{HN} = \frac{HN - WS}{HN} \tag{16}$$

In Equation (15), WS is the solution obtained from the wait and see method in which the first model is solved thirty-two times for all scenarios separately, then the expected value of the total costs that are calculated is

TABLE 8. Optimal PM execution interval obtained from the second model in the presence of all scenarios

$$a^{*}_{I_5=(1,0,0,1,0,0,1)}(s,z)$$

	First inspection	Second inspection					
s	***Z***						
	300	**173**	**211**	**259**	**317**	**389**	**475**
0	3	0	0	2	2	2	2
1	[redacted]	0	0	2	2	2	2
2	[redacted]	0	0	2	2	1	1
3	[redacted]	1	1	1	1	1	1
4	[redacted]	1	1	1	1	1	1

$$a^{*}_{I_{11}=(1,0,1,0,1,0,1)}(s,z)$$

	First inspection	Second inspection			Third inspection								
s	***Z***												
	300	**192**	**288**	**432**	**156**	**190**	**232**	**233**	**285**	**348**	**350**	**428**	**523**
0	[redacted]	2	2	2	0	0	0	0	0	0	0	2	2
1	2	2	2	2	0	0	0	0	0	0	0	2	1
2	2	2	2	1	0	0	0	0	0	1	1	1	1
3	[redacted]	1	1	1	1	1	1	1	1	1	1	1	1
4	[redacted]	1	1	1	1	1	1	1	1	1	1	1	1

$$a^{*}_{I_{21}=(1,1,0,1,0,0,1)}(s,z)$$

	First inspection	Second inspection		Third inspection					
s	***Z***								
	300	**240**	**360**	**173**	**211**	**259**	**317**	**389**	**475**
0	[redacted]	2	2	0	0	2	2	2	2
1	[redacted]	2	2	0	0	2	2	2	2
2	[redacted]	2	2	0	0	2	2	1	1
3	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1

defined as WS solution [22]. Figure 3 shows that the VSS (%) is zero for values of h thatare greater than a large value. This proves that whenthe unit lost sale cost exceeds a threshold, neglecting demand uncertainty in decision making does not affect total cost. Thus, the stochastic solution and solution for the average scenario have the same performance. In addition, the stochastic solution for $h = 35$ in the state 4 with 7.8% has the most reduction in the total costs. Also, Figure 4 shows that the maximum value of EVPI (%) is 4.2%, which indicates that completing the information about future demand decreases the total cost at most 4.2%.

Finally, as an application of the proposed model, it can be applied to the optimization of the maintenance for blades of wind turbines in the offshore wind energy. Blades are a large and expensive part of a wind turbine whose function is to convert the kinetic energy of the wind into mechanical energy. The most common design of wind turbines is with three blades. Blades of wind turbines are typically built with hand-laid fiberglass. As mentioned in literature [24], the deterioration process of blades of the wind turbine can be considered as a continuous Markov process. Also, the demand for electrical energy usually is a stochastic parameter that can be viewed as a scenario-based stochastic parameter. Therefore, According to assumptions of our problem, a wind turbine and its produced electrical energy can be defined as a single-machine and a single-product, respectively. Under this condition, the proposed model leads to an appropriate strategy for the simultaneous planning of inspections and preventive maintenance in the presence of uncertain demand.

TABLE 9. Improvement amount of HN in comparison with another four scenarios solution (%)

s	Scenario			
	Nominal capacity	**Optimistic**	**Average**	**Pessimistic**
0	1.5	11.1	0	1.5
1	9.4	15	0.3	0
2	10.4	17.4	0.3	0
3	4.1	6.3	0	4.1
4	3.5	5.3	0	3.5

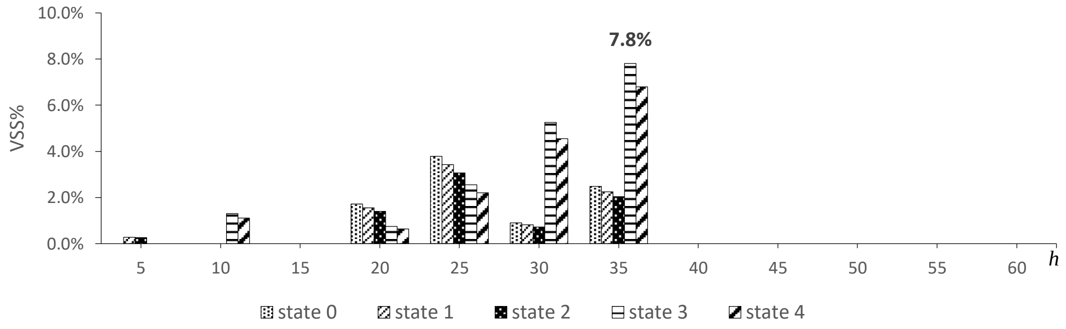

Figure 3. VSS (%) measure in various states for different values of *h*

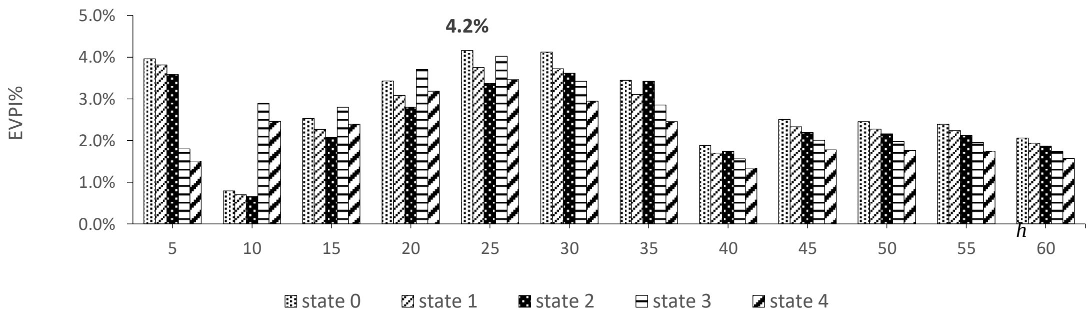

Figure 4. EVPI (%) measure in various states for different values of *h*

6. CONCLUSION

In this paper, a single product single machine system has been studied where the machine deteriorates by a Markov stochastic process. The demand has been considered as a set of scenarios with an arbitrary finite discrete probability distribution in the planning horizon and it has been assumed that there is no action for improvement of the machine during operation and the machine becomes as good as a new machine after PM execution. To integrate the inspection and preventive maintenance planning in a finite time horizon and in a tactical level with considering demand in a certain, the stochastic dynamic process framework has been employed and a model for finding optimal inspection and PM execution interval has been presented whose structure has been depended on the selected inspection plan. The objective of the model is to minimize the sum of inspection, PM, and lost sale costs. In this model, the machine status at the beginning of each period has been considered as the state variable. Then, to analyze the effect of demand uncertainty on decisions and total cost, the second model has been extended. In the second model, the demand has been appended to the state variable of the first model and conditional expected value of the cost given the demand has been replaced with the cost in the first model. In both models, inspection time has been defined as a decision variable and it has been assumed that the value of the state variable is revealed only after inspection execution. For each inspection plan, the corresponding optimality equation has been solved and its results were stored. These results consist of optimal PM execution intervals that are dependent on the corresponding inspection plan. Comparing these results, the optimal inspection plan of both models has been determined. Analysis of numerical results showed that the more the demand and the worse the state of the machine in the inspection time, the more inspection and PM must be done earlier or in the same time. Also, when the value of the unit lost sale is more than the threshold, neglecting demand uncertainty in decision making has no consequence. Two measures VSS (%) and EVPI (%) showed that the use of stochastic solution (HN) and completing the information about future demand decrease the total cost ultimately up to 7.8 and 4.2%, respectively. At the end, as future research, we can point to inserting other decisions of aggregation production planning to the proposed models of this research. Also, establishing sufficient conditions that guarantee the monotonicity in both machine status and demand for the problem with the similar situation in an infinite time horizon can be interesting.

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Persian Abstract

چکیده

در این مقاله یک سیستم تولید تک ماشین و تک محصول در شرایط عدم قطعیت تقاضا در نظر گرفته شده که در آن ماشین طبق یک فرایند مارکفی رو به زوال می‌رود. هدف پیداکردن زمان بهینه برای انجام بازرسی‌ها و نت پیشگیرانه در برنامه‌ریزی نگهداری و تعمیرات مبتنی بر شرایط با بازرسی‌های گسسته است. بدین منظور با استفاده از برنامه‌ریزی پویای تصادفی یک مدل (مدل اول) ارائه شده که متغیر حالت آن، وضعیت ماشین می‌باشد. در این مدل تقاضا قطعی است و هدف، کمینه کردن هزینه‌های بازرسی، نت و فروش از دست رفته می‌باشد. سپس به منظور در نظر گرفتن عدم قطعیت تقاضا، این مدل با استفاده از رویکرد برنامه‌ریزی تصادفی دو مرحله‌ای مبتنی بر سناریو توسعه داده شده است. در مدل توسعه یافته (مدل دوم) انتخاب طرح بازرسی به عنوان تصمیمات مرحله اول و زمان مناسب برای اجرای نت پیشگیرانه به عنوان تصمیمات مرحله‌ی دوم در نظر گرفته شده است. به منظور بررسی تأثیر عدم قطعیت تقاضا، یک مثال طراحی شده که تجزیه و تحلیل نتایج عددی آن نشان می‌دهد اولاً، در زمان بازرسی هر چه تقاضا بیشتر یا ماشین در وضعیت بدتری قرار داشته باشد، تعداد بازرسی‌ها بیشتر و نت پیشگیرانه باید زودتر یا در زمانی مشابه اجرا گردد. ثانیاً، وقتی هزینه‌ی هر واحد فروش از دست رفته از یک حدی بیشتر باشد، نادیده گرفتن عدم قطعیت تقاضا در تصمیم‌گیری هزینه‌ای به همراه ندارد.