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A Mathematical Model for Scheduling Elective Surgeries for Minimizing the Waiting Times in Emergency Surgeries

M. Yazdi*[a], M. Zandieh[b], H. Haleh[c]

[a] *Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *Department of Industrial Management, Management and Accounting Faculty, Shahid Beheshti University, Tehran, Iran*
[c] *Faculty of Industrial Engineering, Golpayegan University of Technology, Golpayegan, Iran*

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A B S T R A C T

The ever-increasing demands for surgeries and the limited resources force hospitals to have efficient management of resources, especially the expensive ones like operating rooms (ORs). Scheduling surgeries including sequencing them, assigning resources to them and determining their start times is a complicated task for hospital managers. Surgery referrals usually include elective surgeries that are admitted before the planning horizon of the schedule and emergency surgeries that arrive during this horizon and require fast services. In this paper, we presented a mathematical model for scheduling electives and emergencies. In our model, we considered surgeries as projects with multi-activities. We implemented the Break-in-Moments (BIMs) technique in this structure, which to our best knowledge has not been implemented in the literature before. We examined this method with real data from a medium-sized Norwegian hospital and observed that this method reduces the waiting time of emergencies to be inserted into the schedule without dedicating any OR merely to emergencies. In such a way, this method counterbalances between efficient OR usage and responsiveness for emergency surgeries.

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1. INTRODUCTION

Healthcare management includes various problems and many researchers are interested in this area. Some of these problems are related to determining the location of healthcare facilities [1, 2] determining the optimum capacity of each facility [3, 4] and healthcare routing problems [5].

One of the important problems, which hospital managers encounter is of the surgery-scheduling problem (SSP), this problem can answer many questions at various strategic, tactical and operational levels of decision-making. These questions change from the selection of surgery specialties to serve in the surgical unit, to dividing the total capacity of operating rooms (ORs) between specialties [6] (OR time blocking) and sequencing the surgeries in a day and assigning the required resources to them [7]. In this paper, we enclose the SSP only in the sequencing of surgeries and assigning the resources to them and determining their start times, which perfectly fits at the operational level. The SSP is well studied in the literature, Cardoen et al. [7] and Zhu et al. [8] classified earlier works in their review papers.

Various mathematical models are considered for the SSP in the literature. A multi-mode blocking job shop that considers the SSP as a flow of patients that moves between different parts in a surgery suite is proposed by Pham and Klinkert [9]. The authors considered each surgery to consist of several activities, moreover, completion of each activity requires several resources. They supposed that each patient moves between these activities and in any activity, the patient consumes some resources. In their model, except in the last activity, patients block the resources of the current activity until moving to the next activity. Jung et al. [10] considered the SSP as a new class of parallel machines scheduling problem. In their paper, each OR is considered as a machine and finally, they proposed an algorithm by the

*Corresponding Author Email: *M.yazdi@qiau.ac.ir* (M. Yazdi)

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integration of Long Processing Time First and Short Processing Time First rules. Another model for the SSP is a multi-project, multi-mode resource-constrained project scheduling problem with generalized precedence relations (MPMRCPS) [11, 12]. This model will be discussed in the following parts of this paper. Aringhieri et al. [13] studied the SSP and the OR time blocks determination problem and proposed a 0-1 linear programming model for this case. The authors discussed that their problem is generally NP-Hard and developed a special metaheuristic for their problem. A mathematical programming model is suggested for a special case of the SSP when unscheduled surgery referrals are deferred to the next planning horizon. Moreover, besides the resources in surgery suite, the availability of the resources in the ward and ICU which are used after and before surgery are also considered [14]. Lamiri et al. [15, 16] proposed a new stochastic programming model for the SSP. Their model schedules surgeries in two phases. The planning phase determines which surgery referrals would be performed in the planning horizon. The later phase determines the assignment of ORs and other resources to surgeries and also sets their start times. Then, for solving real size problems a Monte Carlo optimization approach and a column generation approach are proposed. Monte Carlo optimization is conducted by a mixture of Monte Carlo simulation and mixed-integer optimization method [15]. In the column generation approach, each possible compound of surgeries in an OR is supposed as a column [16]. Hamid et al. [17, 18] in their researches, integrated both phases of planning that are about the selection of the surgeries and scheduling that assigns resources to the surgeries in the SSP together and presented mathematical programming models. The authors considered human personality indicators like personnel's satisfaction and also compatibility among surgical team members in the SSP and presented a mixed-integer multi-objective mathematical model [17].

In this paper, we select MPMRCPS which is less dealt with in the literature, for further development. In this model, every surgery referral is considered as a project of some activities. Various precedence relations between activities of a project can be considered. Each activity can be performed by some sets of resources or some activity modes. For the execution of an activity, the availability of all of the resources in one of its activity modes is required. Moreover, any of the resources has its working hours that only are available in these hours. A feasible schedule equals to sequencing the projects and setting the start time to their activities and assigning the resources to activities with considering some respects (details are illustrated in the following sections of this paper). This is an NP-hard optimization problem [12].

Hospitals are responsible for serving both elective and emergency surgeries. Since electives are admitted before the planning horizon of a schedule, an offline schedule is considered for scheduling of them. However, emergencies are different. Their requests can occur at any time and their urgency forces the hospital to prepare an operating room for them in a limited period. The length of this limited-time period depends on the special case of emergency surgery.

For dealing with emergencies, various OR policies are implemented in hospitals. Some hospitals dedicate some ORs merely to serve emergencies but in some others, ORs are shared between both electives and emergencies or the flexible OR policy is implemented. Moreover, for implementing the flexible OR policy various approaches are applied in the literature. In some papers, some part of total OR time is dedicated to emergencies [19], e. g. emergencies can use OR during 12-14, in an OR with the working hours 8-14. In fact, in this approach, some OR times are dedicated to emergencies and scheduling electives is planned in another part of OR times. In a few papers, another approach for flexible OR policy is implemented, in such a way that emergency surgery can be inserted into the schedule in slack times or instead of one of the current scheduled elective surgeries.These opportunities for inserting emergencies are known as the Break-in-Moments (BIMs) [20, 21]. A special case of implementing the BIMs approach, when each surgery has a stochastic duration with known distribution was studied by Vandenberghe et al. [22].

For further illustration of BIMs, suppose a typical schedule of elective surgeries that are scheduled in a single OR. This OR has only two states, it is free in slack times or it is occupied by an elective surgery. At the arrival of emergency surgery, one of these two states for OR is imaginable. In the first state, the emergency surgery immediately enters the OR, because the OR is free. Otherwise, the emergency surgery has to wait to complete the current elective surgery and after that, it can enter the OR. This is because the surgery is non-interruptible. The problem of inserting an emergency surgery to the schedule of elective surgeries when there is more than one OR is more complex. Figure 1 illustrates the concept of BIMs in a schedule in three ORs. In this example, suppose an emergency surgery that arrives before $BIM1$. All three ORs are occupied while this emergency surgery arrives. $OR1$ is free at $BIM1$, $OR2$ at $BIM2$ and $OR3$ at $BIM3$. Therefore, the first possible opportunity for inserting emergency surgery to the schedule is at $BIM1$ that is earlier than others. Here, emergency surgery enters the $OR1$ instead of the elective surgery ($Surgery2$). The right-hand side of the picture shows the change of the schedule and also BIMs after inserting this emergency surgery to the schedule.

For minimizing the average waiting time of emergencies in the BIMs approach, it is necessary to pay attention to adjusting the distance between successive BIMs, while spreading the elective surgeries between ORs. The problem of adjusting the successive BIM

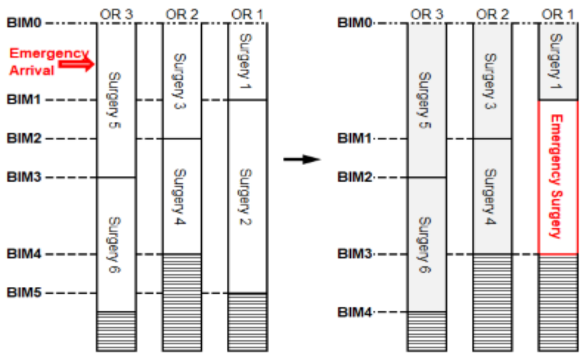

Figure 1. Inserting emergency surgery in the schedule

intervals, when the number of ORs exceeds one, is a strongly NP-hard problem [20].

Various OR policies were discussed above, selecting the best OR policy for dealing with emergency surgeries in a hospital highly depends on the conditions of that hospital and its special scenario [21].

To our best knowledge, the BIMs approach is less dealt with in the literature and implementation of it, when the SSP is modeled as MPMRCPS is the case, which has not been studied before. Therefore, in this paper, we propose a mathematical model for this special case of the SSP. When none of the ORs is dedicated merely to emergencies, our model schedules the electives in ORs by implementing the BIMs approach. This model by adjusting the successive BIM intervals tries to controls the average waiting time in emergency surgeries for inserting to the schedule.

The remainder of this paper is organized as follows. Section 2 provides an overview of the problem description, and the mathematical model of the problem is given in Section 3. Some computational results are illustrated in Section 4. Finally, conclusions and some outlines for future works are addressed in Section 5.

2. PROBLEM DESCRIPTION

In this section, we briefly explain how the SSP is modeled as MPMRCPS. This model for the SSP is introduced in the literature [12] (interested readers can refer to the original reference for more details). Later, we illustrate our main contribution in implementing the BIMs approach in this environment.

Every surgery referral is considered as a project p of some activities N^p. In the sample problem that is solved in our computational results section, a surgery referral consists of surgery, cleaning and recovery activities, $N^p = \{Surgery, Cleaning,\ Recovery\}$. Various precedence relations can be considered, between these activities for describing the upper and lower limits of a patient's waiting time between these activities, e.g. the precedence relation $FS^{max}(15)$ between surgery and cleaning activities in a project p is indicated in this project; cleaning activity can start during a time window between 0 and 15 minutes after finishing the surgery activity.

Execution of each activity $i \epsilon N^p$, in any project p, requires the simultaneous availability of some sets of resources or activity modes. Usually, more than one resource set or activity mode can be chosen for any activity i. For more illustration, consider a typical surgical suite with two ORs (OR_1, OR_2) and two surgeons with the same specialty $(Surgeon_1, Surgeon_2)$ and suppose that surgery activity of project p needs an OR and a surgeon for its execution. In such a way, four activity modes for surgery activity of project p can be considered that are produced by various combinations of ORs and surgeons, (e.g. $(OR_1, Surgery_1)$). In an activity, various modes use different resource sets and thus each of these modes has its own duration time. In our above sample, surgery activity has four different modes and each of these modes has its own duration. This property enables the modeler to consider experimental details in the model, e.g. the difference between the experience of surgeons, and whether the surgery is done in a training operating room or not.

It is supposed that all the projects use a common set of renewable resources R that these resources are from various resource types e.g. ORs, surgeons, recovery roomsand and anesthesiologists. Moreover, there are some resource entities for each resource type e.g. OR_1, OR_2 from resource type OR. Each of resource entities $\forall\, r \in R$ has its own working hours that are considered as a set of availability intervals $K^r = \{k_1, k_2 \dots\}$ and any resource entity is only available with a specified capacity c_q during each of these intervals $k_q \in K^r$. Furthermore, the total capacity of ORs is divided between various surgery specialties (OR time blocking); in any OR block only the surgery referrals from that specialty can access to the OR. For more illustration, suppose that the working hour of OR_1on Mondays is 8-14 and its OR blocks are 8-10 for cardiology and 10-14 for urology; this implies cardiology surgery referrals (projects) can access to this room only during 8-10. We suppose that, these OR blocks are considered only for elective surgeries, but emergency surgeries can access any available OR without attention of OR blocks.

Scheduling a project is equal to determining the start times and selecting an activity mode for all of its activities. Thus, for satisfying the precedence relations between activities, it is required that considering the availability of resources and respecting inter-activity mode compatibility constraints and projects disjunction constraints (including the following details).

Suppose two projects p_1, p_2 that contain two activities of surgery and cleaning, in which surgery is the predecessor of cleaning with relation $FS^{max}(45)$. Also, suppose that, the execution of activity surgery needs a surgeon and an OR for 20 minutes, and execution of activity cleaning needs a cleaner and an OR for 5 minutes. The availability of resources is as follows: OR_1 is available in $[0, 100]$, OR_2 in $[50, 100]$ and $Surgeon_1$ and $Surgeon_2$ are available in $[0, 100]$ and $Cleaner_1$ is available in $[40, 100]$ and each resource has the capacity of one in its availability interval. Suppose p_1 is scheduled before p_2 and the objective function is minimizing the makespan, then the first start time of p_1 is at zero. Activity $Surgery_1$ starts at zero and continues until 20 with resources OR_1 and $Surgeon_1$. Inter-activity mode compatibility constraint means that because both activities, surgery and cleaning in project p_1 use common resource type OR then the same OR entity (OR_1) should be used in the activity $Cleaning_1$. Activity $Cleaning_1$ starts at 40, this is the time that resources OR_1 and $Cleaner_1$ are available, and the precedence relation $FS^{max}(45)$ between activities $Surgery_1$ and $Cleaning_1$ is satisfied. Resource OR_1 between activities $Surgery_1$ and $Cleaning_1$ is free (during $[20, 40]$) but because of the project disjunction constraint, it is quarantined until finishing the last usage of it in the project p_1. This causes project p_2 not to start earlier 45, however, its resources (OR_1, $Surgeon_1$) are available at 20 but due to the last usage of OR_1 in project p_1 is not terminated. The project disjunction constraint prevents usage of OR_1 in other projects until finishing of activity $Cleaning_1$of project p_1 that is the last usage of this common resource in the project p_1.

The existence of common resources in a project results in some precedence relation between activities in a project that uses this common resource. Moreover, all the activities in the current project that use the common resource are predecessors of the activities in other projects that will use this common resource later (due to project disjunction constraints).

Usually in a schedule, scheduling all the projects is not practical due to the restriction of resources, a set of projects are selected for scheduling and this selection is based on an objective function. This mathematical problem is modeled as a mixed-integer linear program [12]. This model has the capability of solving both the resource assignment problem and the sequencing problem as a unified problem in one-step. Moreover, in this model, time is considered as a continuous variable that is a positive point.

Our main contribution is the implementation of the BIMs approach in this model. In all earlier works about the BIMs approach, it is supposed that in each project, the duration of using OR in all ORs is the same and the value of OR usage or its distribution is known. Moreover, they suppose the surgery referrals in an OR or the set of projects that should be scheduled in an OR is given. Then, these methods try to implement the BIM approach, as minimization of the maximum BIM interval. Here, we encounter a special case that the existence of various activity modes in surgery activity of a project makes the earliest approaches of BIMs unsuitable. Because in each project by changing the OR, duration of surgery activity is deferred due to changing the surgery activity mode. As previously discussed, this variation of activity modes comes from considering realistic aspects like the experience of surgeons in the model. Moreover, we also suppose assigning the surgery referrals to ORs is not given before and this assignment is done during the scheduling the projects and adjusting the BIMs intervals. The mathematical model of this problem is discussed in the next section.

3. MATHEMATICAL MODEL

In this section, the mathematical model of the problem and the way of implementing the BIMs approach in the MPMRCPS environment is discussed. Table 1 illustrates the notations used in this model.

TABLE 1. Summary of notations

Notations	Definitions
P	Set of projects (surgery referrals) for scheduling
p	A project (or surgery referral)
N^p	Set of activities in project p
N	Set of all activities in all projects P
M^i	Set of activity modes for activity i
$M^i_{j,m} \subseteq M^i$	Set of feasible activity modes for activity i when activity mode m is selected for activity j
R	Set of resources
R^i	Set of resources assigned to activity i by selecting an activity mode
K^r	Set of availability intervals for resource r
$K^r_i \subseteq K^r$	Set of availability intervals of resource r for activity i
$\mathcal{g}_i$	Set of immediate predecessor activities of activity i
C	Set of resources that may be used as common resources in projects (usually ORs)
S^r_p	Set of activities in project p that do not have predecessor on resource r , $\forall r \in c$
ε^r_p	Set of activities in project p that do not have a follower on resource r, $\forall r \in c$
s^r_k, e^r_k	The start time and finish time of availability interval k of resource r
H	The length of the planning horizon
$[\alpha_i, \beta_i]$	Hard time window constraint for activity i
$[\gamma_p, \epsilon_p]$	Hard time window constraint for project p γ_p(project referral) ϵ_p(upper limit on project completion, or H)
c^r_k	Capacity of resource r in availability interval k

μ_r^m	The value of resource usage of resource r in activity mode m
u_r^m	=1 when mode m uses resource r and 0 otherwise
δ_{ij}	Maximum allowed delay between completion of activity i and start of activity j
ϑ_i^m	Duration of activity i in mode m
ρ^r	Setup time for resource r (only for resources with maximum capacity 1)
φ^k	Starting time of resource availability interval k (hard constraint)
σ^k	Finish time of resource availability interval k (hard constraint)
χ_i^m	=1 if activity i uses activity mode m and 0 otherwise
q_i^k	=1 if activity i uses resource availability interval $k \in K^r$ for resource r and 0 otherwise
t_i	Starting time of activity i
f_{ij}^r	The amount of flow of units of resource r between activity i and activity j
z_{ij}	=1 if activity i precedes activity j and 0 otherwise
$d_{p,p'}^r$	=1 if p precedes p' on r and 0 otherwise, where $r \epsilon C$ and $p, p' \epsilon P$
y_i	Completion time of activity i
C_p	Completion time of project p
d_i^r	The amount of demand for resource r in the activity i
g_i^r	=1 if activity i uses resource r in the chosen mode
h^p	=1 if project p remains unscheduled and 0 otherwise

The objective function of the model is a linear combination of objective components whose minimization is desirable.

$$Min \quad o = \sum w_l O^l \tag{1}$$

In Equation (1), w_l reflects the relative importance of objective component O^l, and objective components are normalized. We consider makespan (Equation (2)) and the number of unscheduled projects (Equation (3)) as objective components.

$$O_M = \frac{1}{H} max_{p \in P} C_p \tag{2}$$

$$O_{unsched} = \frac{1}{|p|} \sum_p h^p \tag{3}$$

Makespan objective component tries to finish the schedule early and unscheduled objective component is used to force as many as projects scheduled in the limited planning horizon.

The problem constraints are as follows:

$$y_i = t_i + \sum_{m \epsilon M^i} \vartheta_i^m x_i^m ; \ \forall \ i \epsilon N \tag{4}$$

$$g_i^r = \sum_{m \epsilon M^i} u_r^m x_i^m ; \ \forall \ r \epsilon R, \forall \ i \epsilon N \tag{5}$$

$$d_i^r = \sum_{m \epsilon M^i} \mu_r^m x_i^m ; \ \forall \ r \epsilon R, \forall \ i \epsilon N \tag{6}$$

$$\sum_{m \epsilon M^i} x_i^m = 1 - h^p \tag{7}$$

$$\sum_{k \epsilon K_i^r} q_i^k = g_i^r ; \ \forall \ i \epsilon N, \forall \ r \epsilon R \tag{8}$$

$$x_j^m \leq \sum_{m' \epsilon M_{j,m}^i} x_i^{m'} ; \ \forall \ i, j \epsilon N^p, \forall \ p \epsilon P \tag{9}$$

$$z_{ij} + z_{ji} \leq 1; \ \forall \ i, j \epsilon N \tag{10}$$

$$z_{ij} = 1; \ \forall \ i \epsilon \mathcal{G}_j, \forall \ j \epsilon N \tag{11}$$

$$z_{ij} \geq (g_i^r + g_j^r - 1); \ \forall \ r \epsilon R, \forall \ (i,j) \epsilon \Pi^r \tag{12}$$

$$t_j - y_i - \rho^r Max(0, g_j^r + g_i^r - 1) \geq (z_{ij} - 1)M; \ \forall i, j \epsilon N, \forall \ r \epsilon R \tag{13}$$

$$d_{p,p'}^r + d_{p',p}^r = 1; \ \forall \ p, p' \epsilon P \tag{14}$$

$$z_{i,j} \geq d_{p,p'}^r + g_j^r + g_i^r - 2; \ \forall r \epsilon C, \forall i \epsilon \varepsilon_p^r, \forall j \epsilon S_{p'}^r, \forall p, p' \epsilon P \tag{15}$$

$$z_{j,i} \geq -d_{p,p'}^r + g_j^r + g_i^r - 1; \ \forall r \epsilon C, \forall i \epsilon S_p^r, \forall j \epsilon \varepsilon_{p'}^r, \forall p, p' \epsilon P \tag{16}$$

$$t_i \geq Max(a_i, \gamma_p); \ \forall p \epsilon P, \forall i \epsilon N^p \tag{17}$$

$$y_i \leq Min(\beta_i, \varepsilon_p); \ \forall p \epsilon P, \forall i \epsilon N^p \tag{18}$$

$$f_{ij}^r \leq z_{ij} M; \ \forall r \epsilon R, \forall i, j \epsilon N \tag{19}$$

$$\sum_{j \epsilon N} f_{ij}^r + \sum_{k \epsilon K_i^r} f_{i\, e_k^r}^r = d_i^r; \ \forall i \epsilon N, \forall r \epsilon R \tag{20}$$

$$\sum_{i \epsilon N} f_{ij}^r + \sum_{k \epsilon K_j^r} f_{s_k^r\, i}^r = d_i^r; \ \forall i \epsilon N, \forall r \epsilon R \tag{21}$$

$$f_{s_k^r\, i}^r \leq c_k^r q_i^k; \ \forall i \epsilon N, \forall k \epsilon K^r, \forall r \epsilon R \tag{22}$$

$$f_{i\, e_k^r}^r \leq c_k^r q_i^k; \ \forall i \epsilon N, \forall k \epsilon K^r, \forall r \epsilon R \tag{23}$$

$$\sum_{i \epsilon N} f_{s_k^r\, i}^r + f_{s_k^r\, e_k^r}^r = c_k^r; \ \forall k \epsilon K^r, \forall r \epsilon R \tag{24}$$

$$\sum_{i \epsilon N} f_{i\, e_k^r}^r + f_{s_k^r\, e_k^r}^r = c_k^r; \ \forall k \epsilon K^r, \forall r \epsilon R \tag{25}$$

$$t_i - \varphi^k q_i^k \geq 0; \ \forall i \epsilon N, \forall r \epsilon R, \forall k \epsilon K_i^r \tag{26}$$

$$y_i - \sigma^k q_i^k - (1 - q_i^k)M \leq 0; \ \forall i \epsilon N, \forall r \epsilon R, \forall k \epsilon K_i^r \tag{27}$$

$$t_j - y_i - \delta_{ij} \leq 0; \ \forall i, j \epsilon N \tag{28}$$

$$z_{ij} - z_{ji} \geq g_i^r + g_j^r - 1;\ \ \forall i, j \epsilon N \tag{29}$$

$$x_i^m \epsilon \{0, 1\};\ \forall\ i\epsilon N, \forall\ m\epsilon M^i \tag{30}$$

$$z_{ij} \epsilon \{0, 1\};\ \ \forall\ i, j\epsilon N \tag{31}$$

$$t_i \epsilon \mathbb{R}_+;\ \ \forall\ i\epsilon N \tag{32}$$

$$q_i^k \epsilon \{0, 1\};\ \forall\ r\epsilon R, \forall\ k\epsilon K_i^r \tag{33}$$

$$f_{ij}^r \epsilon \mathbb{R}_+;\ \ \forall\ r\epsilon R, \forall\ i, j\epsilon N \cup (\cup_{k\epsilon K^r} \{s_k^r, e_k^r\}) \tag{34}$$

$$d_{p,p'}^r \epsilon \{0, 1\};\ \forall\ r\epsilon \complement, \forall\ p, p'\epsilon P \tag{35}$$

$$h^p \epsilon \{0, 1\};\ \forall\ p\epsilon P \tag{36}$$

$$y_i \epsilon \mathbb{R}_+;\ \forall\ i\epsilon N \tag{37}$$

$$g_i^r \epsilon \{0, 1\};\ \forall\ r\epsilon R, \forall\ i\epsilon N \tag{38}$$

The explanation of the above constraints are as follows:

Equation (4) sets the termination time of activity i as the summation of its start time and its duration. The initialization of variables g_i^r and d_i^r are done in Equations (5) and (6). The binary variable g_i^r takes the value one when activity i uses resource r in Equations (5) and variable d_i^r takes the amount of demand for resource r in the activity i in Equations (6). Equation (7) is used to force exactly the selection of one mode for each activity in any scheduled project. Equation (8) is used to ensure when activity i uses resource r then exactly one of the availability intervals of resource r should be selected. Equation (9) is responsible to establish the consistency between activity modes in any project, and Equation (10) prevents both of any two activities from becoming predecessor of each other. Equation (11) makes any immediate predecessor of any activity i as its precedence activity. Equation (12) makes one of the activities i, j as the predecessor of the other one when both of them use a common resource. Equation (13) sets the start time of activity greater than the finish time of its following activities plus the setup time of the common resource. Equation (14) forces one of any two projects that use the same common resource to become the precedence of another one. Equations (15) and (16) build a precedence relation between the last activity that uses a common resource in project p and the first activity that uses the same common resource in project p' when project p starts earlier than project p' in the schedule. Equation (17) sets the start time of each activity in any project after the first referral of that project and the first start time of hard time window of the activity. Equation (18) sets the finish time of each activity in any project before the last completion time of the project and the last finish time of hard time window of the activity. Equation (19) restricts the flow of resources between two activities to the existence of precedence relation between these activities. Equations (20) and (21) set the flow of resources between the resource availability intervals to each activity and set the flow of resources between activities based on resource demands. Equations (22)-(25) altogether are responsible to make the flow of resources between resource availability intervals and activities based on the capacity of resources in resource availability intervals. Equations (26) and (27) make the start time and finish time of activity lie in the resource availability intervals of its resources. Equation (28) controls the maximum delay between activities in a project. Equation (29) is another constraint to control the precedence relation between two activities with a common resource. Finally, Equations (29)-(38) are about defining the variables.

To implement the BIM approach, the minimization of the maximum distance between two successive BIMs interval is demanded. As discussed in the previous section, in earlier works, it is supposed that the surgeries, which are assigned to an operating room, are given and the duration of using operating rooms for each surgery or the distribution of this time is known.

Here, we encounter a special structure of the problem that each surgery referral is a project with multiple activity modes, thus the duration of using operating rooms depends on activity modes. Moreover, we are interested to remove the assumption of assigning the projects to operating rooms before implementing the BIMs approach. We use a heuristic by adding a virtual activity to each project. In any project, just after the start of surgery activity until a limited time, the virtual activity can start. This limited time is considered as the average durable waiting time for emergencies that is usually a given value. We suppose the duration in virtual activities as the average of the estimated time of OR usage in emergency surgeries (that is a known value) and their resource requirements are just an OR. Since this is a virtual activity, the resource OR in this activity is not considered in inter-activity mode compatibility constraints. Then, we consider the previous mathematical model for these elective surgeries with these changes. In the next section, we present the result of solving a sample problem with this method.

4. COMPUTATIONAL RESULTS

In this section, at first, we show how our model works with a small sample problem then, we give the results of solving a real problem based on the presented method.

Suppose a case in which each project consists of three activities (surgery, recovery, and cleaning) and surgery activity is the predecessor of two other activities. Recovery can start up to 10 minutes after termination of surgery ($FS^{max}(10)$) and cleaning can start up to 5 minutes after termination of surgery ($FS^{max}(5)$). The details of three projects and their activity modes are given in Table 2.

It is supposed that all the resources are available from 8 a.m. to 3 p.m. Midnight is considered as start time and time is considered as a continuous variable, so the availability of resources is as follows: operating rooms and surgeons are available from 480 to 900 (or 8 a.m. to 3 p.m.) with capacity one and recovery room and cleaner are available in the same interval with capacity two.

Then, two feasible schedules are considered. The first one that schedules projects without respecting the adjustment of BIM intervals (method A) instead of the second one that schedules projects by considering the adjustment of the BIM intervals (method B). In both of these schedules, the sequence of projects is supposed $(Project_3, Project_2, Project_1)$. Details of these schedules are illustrated below.

Figure 2 represents details of resource consumption when method A of scheduling is selected.

In $Project_3$ by selecting mode 4 for surgery activity, resources $OR_2, Gastro_2$ are demanded and because these resources are free and available, then these resources are occupied from 480 until 588. After surgery, recovery activity starts and this activity occupies one unit of recovery room from 588 to 674. Moreover, cleaning activity can also start after surgery, cleaning requires an OR and a cleaner, but due to inter-activity mode compatibility constraint, only mode 2 of this activity can be selected that uses the same resource OR_2 (the same common resource that is used in activity surgery of this project). In the same way, $Project_1$ starts at 480 when, mode 1 in surgery activity is selected, and then this activity consumes one unit of resources $OR_1$and $Gastro_1$ from 480 to 588. In the case of selection modes 3 or 4 of this activity, the start time of$Project_1$ cannot be earlier than 603, that is because of projects disjunction constraints (the termination of usage resource OR_2 in $Project_3$). Moreover, in the case of selecting mode 2 in surgery activity, this project cannot start earlier than 588, that is because of the occupation of $Gastro_2$ in $Project_1$.

After surgery, other activities in this project are executed as in Figure 2 and this project terminates at 718.

In $Project_2$, mode 4 of surgery activity that uses OR_2 and $Gastro_2$ starts at 603 and by continuing the similar way as previous projects, finally this project terminates at 716.

TABLE 2. Projects specification

Project	Activity	Activity mode	Resource requirements	Duration (minutes)
$Project_1$	$Surgery$	1	$OR_1, Gastro_1$	223
		2	$OR_1, Gastro_2$	177
		3	$OR_2, Gastro_1$	213
		4	$OR_2, Gastro_2$	258
	$Recovery$	1	$Recovery\ Room$	37
	$Cleaning$	1	$OR_1, Cleaner$	15
		2	$OR_2, Cleaner$	15
$Project_2$	$Surgery$	1	$OR_1, Gastro_1$	118
		2	$OR_1, Gastro_2$	91
		3	$OR_2, Gastro_1$	95
		4	$OR_2, Gastro_2$	98
	$Recovery$	1	$Recovery\ Room$	59
	$Cleaning$	1	$OR_1, Cleaner$	15
		2	$OR_2, Cleaner$	15
$Project_3$	$Surgery$	1	$OR_1, Gastro_1$	167
		2	$OR_1, Gastro_2$	155
		3	$OR_2, Gastro_1$	156
		4	$OR_2, Gastro_2$	108
	$Recovery$	1	$Recovery\ Room$	86
	$Cleaning$	1	$OR_1, Cleaner$	15
		2	$OR_2, Cleaner$	15

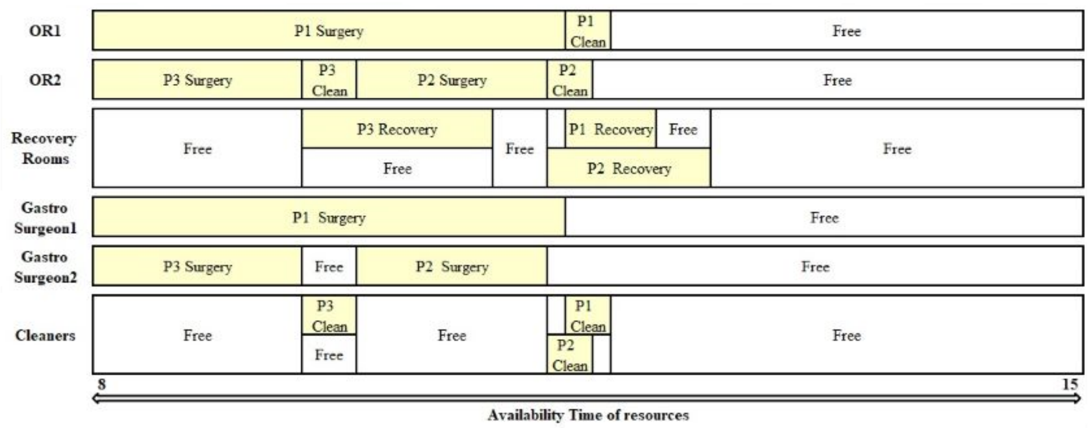

Figure 2. Scheduling of projects without adjusting the BIM intervals (method A)

Figure 3 shows the same projects with the same sequence that are scheduled by method B. Here $Project_3$ is scheduled similar to the previous schedule that is conducted by method A. In the case $Project_1$, resource $OR_1$is available after 480, if this project starts at 480 then the usage of OR_1 in this project will last until 718 this means that the next BIM will be at 603 (usage of OR_2 in $Project_3$ terminates at 603 and $603 < 718$). This makes the lenght of the BIM interval ($603 - 480 = 123$) longer than 50 (the predefined BIM interval), thus $Project_1$ can not start at 480 in OR_1. It is supposed that the average estimated time of OR usage in emergency surgeries is 60 minutes. Thus, we consider an artificial activity in $Project_3$ whose start time can be from 480 (the start of surgery activity) up to 50 minutes and its duration is 60 minutes. This makes the earliest start time of surgery activity in $Project_1$ at 520 ($480 + 60$), we arbitrarily select mode 2 for this activity that requires the availability of $Gastro_2$, it causes this project to start at 588. In a similar way, an artificial activity in $Project_1$can start after the strat time of surgery activity (588) up to 50 minutes later with a duration of 60 minutes. This activity starts at 603 after termination of usage OR_2 in $Project_3$, this artificial activity ends at 663. The earliest start time of $Project_2$ in OR_2 is after 663 when surgery activity in mode 3 can start. Other activities of this project are scheduled in an ordinary way.

To compare the effectiveness of the methods A and B in scheduling of projects, we use the real data from a medium Norwegian hospital, that is available on the web [23]. The file CaseW40-1.xml is used as the information of elective surgery referrals. In this file, the information about the availability of resources and details of 40 surgery referrals are given. The planning horizon is a week and information of the availability of following renewable resources is given: four ORs, recovery room with the capacity of 18 patients, surgeons (consisting of three gastrologist, two urologists and two cardiologists) and three cleaners.

Each project consists of surgery, cleaning, and recovery activities. Surgery activity needs a surgeon and an OR, cleaning activity needs an OR and a cleaner and finally, recovery activity needs a recovery room. In general, surgery activity is the predecessor of two other activities but, cleaning and recovery activities can start simultaneously. Each activity in any project can be executed with some modes, and has a specific duration in each mode. File CaseW40-1.xml gives the details of activity modes in all the projects. We consider file CaseW40-2.xml as a source of emergency surgery referrals for detecting some estimations about emergencies, based on the information of this file, we suppose the average usage of OR in emergency surgeries is 100 minutes. It is supposed that the average durable waiting time in emergency surgeries (the predifined BIM interval) is 60 minutes. All 40 elective projects are scheduled two times, first by method A or without adding the artificial activity (without adjusting the BIM intervals) and second by method B or by adding an artificial activity (with adjusting the BIM intervals). These schedules are obtained by coding the mathematical model in the previous section with Visual C++ environment and IBM ILOG CPLEX Optimization Studio with Concert technology and run on a system with Intel Core i7, 2.2 GHz processor and 8 GB RAM.

In the next step, we select eight projects from file CaseW40-2.xml randomly as emergency projects. The number of emergency surgeries (eight) is considered as 20 percent of the number of elective surgeries (40). According to the research of Bowers and Mould [24], the number of emergency surgeries usually depends on the number of elective surgeries. As they are shown, the number of emergency surgeries is usually about 25 percent of the number of elective surgeries even in anorthopedic department. After selection, the emergency projects, eight random arrival times for them are generated by a Poisson process. Since our planning duration is, a week and ORs are only available during

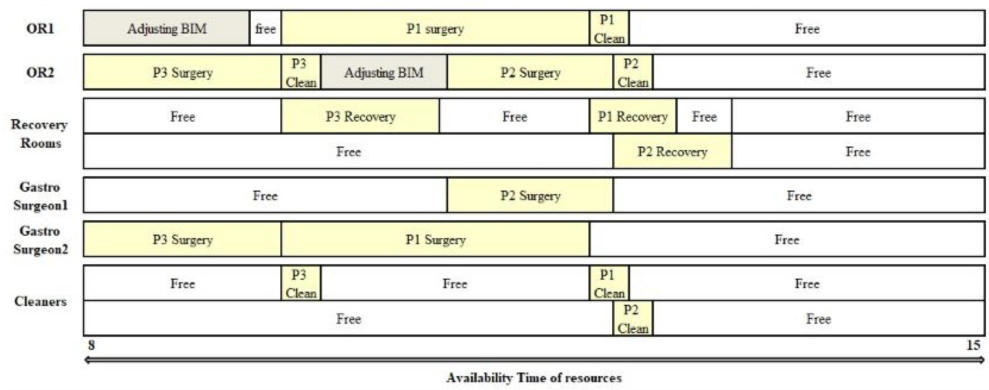

Figure 3. Scheduling of projects with adjusting the BIM intervals (method B)

some working hours, these arrival times are adjusted in such a way that fit only in working hours of ORs in a week. Then, in order to compare the efficiency of two methods A and B, these eight emergency projects with respect of their corresponding arrival times are inserted into the initial schedule that is built from scheduling 40 elective projects with method A. The same experiment is repeated for inserting the emergency projects to the schedule of 40 elective projects with method B. In both of the experiments, the sum of waiting times for emergency projects for inserting into the schedule is calculated.

The process of selection of emergency projects and inserting them into the schedules of elective projects in two methods and obtaining the sum of waiting times in emergency projects is repeated 10 times. Table 3 represents the results of the following efficiency measurements in these experiments: the sum of waiting times for emergency projects (R1), objective functions (R2), number of unscheduled projects (including both groups of elective projects and emergency projects) and number of unscheduled emergency projects in the planning horizon (R3) and the number of unscheduled emergency projects in the planning horizon (R4).

We utilized the statistical software SAS 9.2 for analyzing the results. Tukey test with the confidence interval of 95% is used for comparing the mean of efficiency measurements between pairs of response variables from two methods. Based on the results of Tukey test, in method B the mean of response variable R1 (the sum of waiting times of emergency projects to insert to the schedule) is less than this value in method A. However, Tukey test shows that the mean of response variable R2 (the objective function) in method A is less than this mean in method B. About the mean of the response variable R3 (the number of unscheduled projects including both groups of elective and emergency projects in the planning horizon), in method A is less than this value in method B. Although there is no significant difference between the mean of the response variable R4 (the number of unscheduled emergency projects) in both methods of A and B according to the Tukey test.

TABLE 3. The result of the response variables in the experiments

No.	Project numbers that are selected as emergency projects from file CaseW40-2.xml	R1: Sum of waiting times of emergency projects to insert to the schedule of 40 elective projects		R2: The value of objective function		R3: The number of unscheduled projects		R4: The number of unscheduled emergency projects	
		Method A: scheduling of elective projects without adjusting the BIM intervals	Method B: scheduling of elective projects with adjusting the BIM intervals	Method A	Method B	Method A	Method B	Method A	Method B
1	8, 22, 40, 14, 3, 28, 38, 39	53.14	48.17	0.49	0.51	2	2	1	2
2	38, 25, 18, 26, 28, 32, 14, 30	55	13.17	0.53	0.53	4	5	2	1
3	28, 1, 21, 37, 38, 16, 9, 39	77.14	29.14	0.52	0.54	3	5	2	2
4	31, 7, 8, 15, 38, 39, 1, 2	51.5	34.14	0.50	0.53	2	4	2	2
5	10, 40, 14, 11, 16, 29, 27, 30	36.5	16.83	0.49	0.55	1	5	0	0
6	5, 4, 18, 39, 40, 7, 28, 38	62.83	33.83	0.53	0.52	4	4	2	1
7	37, 7, 11, 19, 5, 27, 29, 35	39.33	37.33	0.51	0.53	3	4	1	1
8	3, 21, 27, 28, 20, 33, 38, 10	55.67	43.83	0.51	0.52	2	4	2	1
9	11, 27, 6, 19, 4, 23, 5, 12	55.83	62.17	0.50	0.52	2	4	1	1
10	2, 9, 13, 38, 22, 36, 39, 7	57.57	46.5	0.50	0.52	1	4	1	1

In other words, adjusting the BIM intervals during the schedule of elective projects is a successful method for decreasing the waiting time in emergency projects. Although, the cost of this success is paid by longer makespan (finishing the schedule later) and increasing the number of unscheduled elective projects, when this method is compared by ordinary scheduling of elective projects. Decreasing the waiting time for emergency surgeries is a very important result because of the responsibility of the hospitals in saving lives. On the other hand, this is considered that this success is obtained while all the ORs are utilized for serving the elective surgeries. In such a way, the profit-making aspect of the ORs is also considered. These results are obtained when arrival rate of emergency surgeries is supposed a normal rate of 20 percent of the number of elective surgeries. However, when this rate increases significantly and hospital encounters lots of emergency surgery referrals, dedicating some ORs to emergencies is suggested.

5. CONCLUSION

In this paper, we introduced a new way of implementing the method of adjusting the BIM intervals for project scheduling model and this is our main contribution. We examined our method with the real data from a Norwegian hospital. This method is successful for decreasing the waiting times of emergency projects for inserting to the schedule of elective projects. However, this method increases the number of unscheduled elective projects that is the drawback of this method. The importance of decreasing the waiting time in emergency projects without dedicating any OR to emergency projects and withdrawing this OR from serving elective projects can be a sufficient reason for making this method attractive for OR managers. We propose using this method in surgery units with very high-profit ORs and low rate of emergency arrivals.

In this research, we suppose that after the arrival of any surgery referral, an expert based on his or her previously experiments determines the feasible modes for activities and their duration times. Failure of resources and unpredictable unavailability of resources and changing the duration of activity modes are not considered in this paper. As future work, we suggest covering more uncertainties in this model.

6. REFERENCES

1. Ahmadi-Javid, A., Seyedi, P. and Syam, S., "A survey of healthcare facility location", ***Computers & Operations Research***,Vol. 79, (2017), 223-263.
2. Zarrinpoor, N., Fallahnezhad, M. S. and Pishvae, M. S., "Design of a Reliable Facility Location Model for Health Service Networks", ***International Journal of Engineering, Transactions A: Basics***, Vol. 30, (2017), No. 1, 75-84.
3. Pantzartzis, E., Edum-Fotwe, F. and Price, D., "Sustainable healthcare facilities: Reconciling bed capacity and local needs", ***International Journal of Sustainable Built Environment***, Vol. 6, No. 1, (2017), 54-68.
4. Maleki Rastaghi, M., Barzinpour, F. and Pishvaee, M. S., "A Multi-objective Hierarchical Location-allocation Model for the Healthcare Network Design Considering a Referral System", ***International Journal of Engineering, Transactions B: Applications***, Vol. 31, No. 2 , (2018), 365-373.
5. Fathollahi-Farda, A. M., Hajiaghaei-Keshteli, M. and Tavakkoli-Moghaddam, R., "A Lagrangian Relaxation-based Algorithm to Solve a Home Health Care Routing Problem", ***International Journal of Engineering, Transactions A: Basics***, Vol. 31, No. 10, (2018), 1734-1740.
6. Shafaei, R. and Mozdgir, A., "Case Mix Planning using The Technique for Order of Preference by Similarity to Ideal Solution and Robust Estimation: a Case Study", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 30, No. 9, (2017), 1362-1371.
7. Cardoen, B., Demeulemeester, E. and Beliën, J., "Operating room planning and scheduling: a literature review", ***European Journal of Operational Research,*** Vol. 201, No. 3, (2010), 921–932.
8. Zhu, S., Fan, W., Yang, S., Pei, J. and Pardalos, P., "Operating room planning and surgical case scheduling: a review of literature", ***Journal of Combinatorial Optimization***, Vol. 37, No. 3, (2019), 757–805.
9. Pham, D. and Klinkert, A., "Surgical case scheduling as a generalized job shop scheduling problem", ***European Journal of Operational Research***, Vol. 185, (2008), 1011-1025.
10. Jung, K., Pinedo, M., Sriskandarajah, C. and Tiwari, V., "Scheduling Elective Surgeries with Emergency Patients at Shared Operating Rooms", ***Production and Operations Management***, Vol. 28, No. 6, (2019), 1407–1430.
11. Riise, A., Mannino, C. and Burke, E. K., "Modelling and solving generalized operational surgery scheduling problems", ***Computers and Operations Research***, Vol. 66, (2016), 1-11.
12. Riise, A. and Mannino, C. "The surgery scheduling problem - a general model", Report SINTEF, (2012).
13. Aringhieri, R., Landa, P., Soriano, P., Tànfani, E. and Testi, A., "A twolevel metaheuristic for the operating room scheduling and assignment problem", ***Computers and OperationsResearch***, Vol. 54, (2015), 21-34.
14. Moosavi, A. and Ebrahimnejad, S., "Scheduling of Elective Patients Considering Upstream and Downstream Units and Emergency Demand Using Robust Optimization", ***Computers & Industrial Engineering***, Vol. 120, , (2018), 216-233.
15. Lamiri, M., Xie, X., Dolgui, A. and Grimaud, F. "A stochastic model for operating room planning with elective and emergency demand for surgery", ***European Journal of Operational Research***, Vol. 185, (2008), 1026-1037.
16. Lamiri, M., Xie, X. and Zhang, S., "Column generation approach to operating theater planning with elective and emergency patients", ***IIE Transactions***, Vol. 40, No. 9, (2008), 838-852.
17. Hamid, M., Nasiri, M.M., Werner, F., Sheikhahmadi, F., and Zhalechian, M., "Operating room scheduling by considering the decision-making styles of surgical team members: A comprehensive approach", ***Computers and Operations Research***, Vol. 108, (2019), 166-181.
18. Hamid, M., Hamid, M., Nasiri, M.M. and Ebrahimnia, M., "Improvement of operating room performance using a multi-objective mathematical model and data envelopment analysis: a

case study", ***International Journal of Industrial Engineering & Production Research,*** Vol. 29, No. 2, (2018), 117-132.

19. Ferrand, Y., Magazine, M. and Rao, U., "Managing operating room efficiency and responsiveness for emergency and elecrive surgeies-A literature survey", ***IIE Transacions on Healthcare Systems Engineering***, Vol. 4, (2014), 49-64.

20. van Essen, J., Hans, E., Hurink, J. and Oversberga, A., "Minimizing the waiting time for emergency surgery", ***Operations Research for Health Care***, Vol. 1, No. 2-3, (2012), 34-44.

21. Duma, D. and Aringhieri, R., "The management of non-elective patients: shared vs. dedicated policies", ***Omega***, Vol. 83, (2019), 199-212.

22. Vandenberghe, M., Vuyst, S. and Aghezzaf, E., "Surgery Sequencing to Minimize the Expected Maximum Waiting Time of Emergent Patients", ***European Journal of Operational Research***, Vol. 275, No. 3, (2019), 971-982.

23. Stølevik, M., Nordlander, T. and Riise, A. "Nurse Rostering Test Cases", SINTEF ICT Available from: https://www.sintef.no/Projectweb/Health-care-optimization/Testbed/ (2010), Accessed 3 November 2016.

24. Bowers, J. and Mould, G., "Managing uncertainty in orthopedic trauma theaters", ***European Journal of Operational Research***, Vol. 154, No. 3, (2004), 599-608.

A Mathematical Model for Scheduling Elective Surgeries for Minimizing the Waiting Times in Emergency Surgeries

M. Yazdi[a], M. Zandieh[b], H. Haleh[c]

[a] *Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *Department of Industrial Management, Management and Accounting Faculty, Shahid Beheshti University, Tehran, Iran*
[c] *Faculty of Industrial Engineering, Golpayegan University of Technology, Golpayegan, Iran*

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چکیده

تقاضای روز افزون جراحی ها و محدودیت منابع، بیمارستان ها را به مدیریت کارآمدی روی منابع خود مخصوصا منابع گران ماانند اتاق عمل ها واداشته است. زمان بندی جراحی ها شامل مرتب کردن آنها، تخصیص منابع و تعیین زمان های شروع آنها یکی از وظایف پیچیده ی مدیر بیمارستان است. ارجاعات جراحی شامل جراحی های انتخابی است که قبل از دوره زمان بندی پذیرش می شوند و همچنین جراحی های اورژانسی که معمولا در حین دوره زمانبندی پذیرش می شوند و نیاز به سرویس دهی سریع دارند. در این مقاله ما یک مدل ریاضی برای مساله زمان بندی جراحی های انتخابی و اورژانسی ارائه کرده ایم. در این مدل جراحی ها به صورت پروژه هایی با چندین فعالیت در نظر گرفته شده اند. ما تکنیک لحظات گسست (BIMs) را در این ساختار پیاده کرده ایم، که تا آنجاییکه ما می دانیم چنین کاری قبلا در ادبیات انجام نشده است. ما این روش را با داده های واقعی یک بیمارستان نروژی با سایز متوسط امتحان کردیم و مشاهده کردیم که این روش قادر است زمان انتظار در اورژانسی ها برای ورود به زمان بندی را کاهش دهد بدون اینکه هیچ اتاق عملی را صرفا به اورژانسی ها تخصیص دهد. بدین نحو، این روش بین استفاده کارآمد از اتاق عمل ها و مسئولیت پذیری در خصوص جراحی های اورژانسی تعادل برقرار می کند.

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